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(be they represented as a full joint distribution or belief network)
- Each world ω has a probability $P(\omega)$
- Taking **any** sentence ϕ we can compute its probability:

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- **Problem?**

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- **Problem?**
 - FOL introduces an infinite set of possible worlds!
 - Possible solution:
unique names assumption + domain closure = **database semantics**

RELATIONAL PROBABILITY MODELS (RPM)

- Database semantics (ensures the finiteness of possible worlds)
 - Except that the closed world assumption is eliminated
 - Probabilistically it does not make sense that all unknown fact are false!
- RPM have constants, functions, and predicates (considered Boolean functions)
- Each function has a **type signature**

$$\begin{aligned} \text{Honest} &: \text{Customer} \longrightarrow \{\text{true}, \text{false}\} \\ \text{Kindness} &: \text{Customer} \longrightarrow \{1, 2, 3, 4, 5\} \\ \text{Quality} &: \text{Book} \longrightarrow \{1, 2, 3, 4, 5\} \\ \text{Recommendation} &: \text{Customer} \times \text{Book} \longrightarrow \{1, 2, 3, 4, 5\} \end{aligned}$$

- Random variables are obtained by the instantiating each function with each possible argument
 - Each type has finitely many instances \Rightarrow number of random variables is finite

RPM (CONT'D)

- Dependencies between random variables are stated as one dependency statement for each function

$$\begin{aligned} \text{Honest}(c) &\sim \langle 0.99, 0.01 \rangle \\ \text{Kindness}(c) &\sim \langle 0.1, 0.1, 0.2, 0.3, 0.3 \rangle \\ \text{Quality}(b) &\sim \langle 0.05, 0.2, 0.4, 0.2, 0.15 \rangle \\ \text{Recommendations}(c, b) &\sim \text{RecCPT}(\text{Honest}(c), \text{Kindness}(c), \text{Quality}(b)) \end{aligned}$$

- *RecCPT* is a conditional distribution with $2 \times 5 \times 5$ rows
- Conditional expressions are possible:

$$\text{Recommendations}(c, b) \sim \begin{cases} \text{if } \text{Honest}(c) \text{ then} \\ \quad \text{HonestRecCPT}(\text{Kindness}(c), \text{Quality}(b)) \\ \text{else } \langle 0.4, 0.1, 0.0, 0.1, 0.4 \rangle \end{cases}$$

just a more compact way of representing the conditional distribution *RecCPT*

- Instantiate these dependencies \Rightarrow a belief network = **the semantics of the RPM**

- Eliminating the closed world assumption
 - Real-life problem: **relational uncertainty**
 - How can we ascertain that $Fan(C1, Author(B1))$ if the author of $B1$ is unknown?
 - * We reason about all the possible authors!
 - * Suppose there are n possible authors $A1, \dots, An$
 - * Then $Author(B1)$ is a random variable with possible values $A1, \dots, An$
- Other conditions:
 - No dependency must be cyclic (since a belief network cannot have cycles)
 - Recursive dependencies are not supported (since this will generate infinite paths in the belief network)

- **Unrolling:**
 - Collect constants and evidence
 - Construct the dependencies
 - Build the associated belief network
 - Apply inference in the belief network
- The resulting networks are very large
 - The usual solution is to construct the network on the fly, not at the beginning
 - Many of the factors constructed during variable elimination will be identical
 - * Efficient caching of previous results improves the algorithm dramatically

OTHER APPROACHES TO REASONING WITH UNCERTAIN DATA

- Major apparent discrepancy between our mind (qualitative) and the probability theory (quantitative)
 - However, no better solution is known
- Dealing with **ignorance**: interval-valued degrees of belief (the Dempster-Shafer theory)
- **Fuzzy logic** allows vagueness (a sentence can be “sort of” true)
 - Vagueness and uncertainty are however orthogonal issues

FUZZY LOGIC

- Is Jim tall? It depends. If he is around 180cm tall, then many people will hesitate
- Instead we can recognize that there are degrees of tallness:
 - the truth value of $Tall(Jim)$ is a number between 0 and 1 instead of just true or false
 - Generally to every fact A we assign a degree of truth $T(A)$ (between 0 and 1)– T is the **fuzzy truth** function
- Once the truth value of facts is known the truth value of complex sentences can be established inductively:

$$\begin{aligned} T(A \wedge B) &= \min(T(A), T(B)) \\ T(A \vee B) &= \max(T(A), T(B)) \\ T(\neg A) &= 1 - T(A) \end{aligned}$$

- All the inference methods work well, but there are problems with relating fuzzy truth with reality:

$$T(Tall(Jim) \wedge \neg Tall(Jim)) = 0.4 ???$$