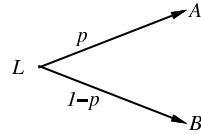


PREFERENCES

- An agent chooses among **prizes** (A, B , etc.) and **lotteries**, i.e., situations with uncertain prizes

Lottery $L = [p, A; (1 - p), B]$



- Notation:

$A \succ B$ A preferred to B
 $A \sim B$ indifference between A and B
 $A \not\succeq B$ B not preferred to A

RATIONAL PREFERENCES

- Idea: preferences of a rational agent must obey constraints.
 - Rational preferences \Rightarrow behavior describable as maximization of expected utility

- Constraints:

- Orderability:** $(A \succ B) \vee (B \succ A) \vee (A \sim B)$
- Transitivity:** $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$
- Continuity:** $A \succ B \succ C \Rightarrow \exists p [p, A; 1 - p, C] \sim B$
- Substitutability:** $A \sim B \Rightarrow [p, A; 1 - p, C] \sim [p, B; 1 - p, C]$
- Monotonicity:** $A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; 1 - p, B] \succsim [q, A; 1 - q, B])$

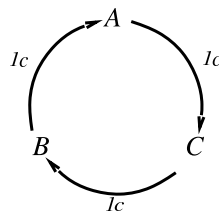
RATIONAL PREFERENCES (CONT'D)

- Violating the constraints leads to self-evident irrationality
- For example: an agent with intransitive preferences can be induced to give away all its money

If $B \succ C$, then an agent who has C would pay (say) 1 cent to get B

If $A \succ B$, then an agent who has B would pay (say) 1 cent to get A

If $C \succ A$, then an agent who has A would pay (say) 1 cent to get C



MAXIMIZING EXPECTED UTILITY

- Theorem** (Ramsey, 1931; von Neumann and Morgenstern, 1944): Given preferences satisfying the constraints there exists a real-valued function U such that

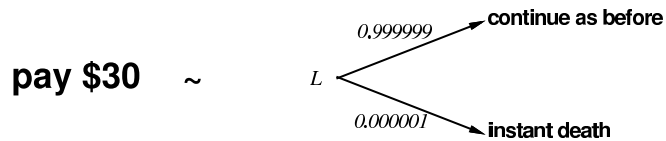
$$U(A) \geq U(B) \Leftrightarrow A \succsim B$$

$$U([p_1, S_1; \dots; p_n, S_n]) = \sum_i p_i U(S_i)$$

- MEU principle:** Choose the action that maximizes expected utility
- Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
 - E.g., a lookup table for perfect tictactoe

UTILITIES

- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment of human utilities:
 - compare a given state A to a **standard lottery** L_p that has
 - * “best possible prize” u_{\top} with probability p
 - * “worst possible catastrophe” u_{\perp} with probability $(1 - p)$
 - adjust lottery probability p until $A \sim L_p$



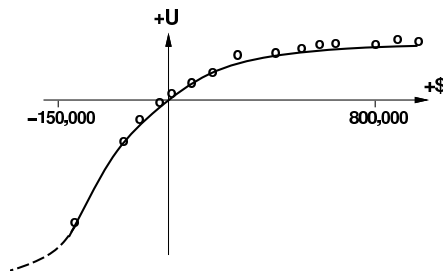
UTILITY SCALES

- **Normalized utilities:** $u_{\top} = 1.0, u_{\perp} = 0.0$
- **Micromorts:** one-millionth chance of death
 - useful for Russian roulette, paying to reduce product risks, etc.
- **QALYs:** quality-adjusted life years
 - useful for medical decisions involving substantial risk
- Note: behavior is **invariant** w.r.t. positive linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$
- With deterministic prizes only (no lottery choices), only **ordinal utility** can be determined, i.e., total order on prizes

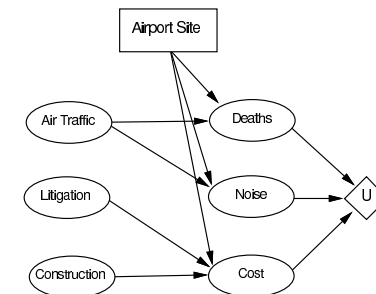
MONEY

- Money does **not** behave as a utility function
- Given a lottery L with expected monetary value $EMV(L)$, usually $U(L) < U(EMV(L))$, i.e., people are **risk-averse**
- Utility curve: for what probability p am I indifferent between a fixed prize x and a lottery $[p, \$M; (1 - p), \$0]$ for large M ?
- Typical empirical data, extrapolated with **risk-prone** behavior:



DECISION NETWORKS

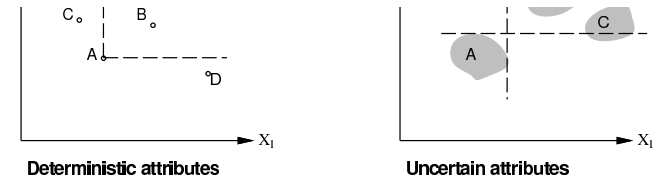
- Add **action nodes** and **utility nodes** to belief networks to enable rational decision making



- Algorithm:
 - For each value of action node
 - compute expected value of utility node given action, evidence
 - Return MEU action

- How can we handle utility functions of many variables $X_1 \dots X_n$? E.g., what is $U(\text{Deaths}, \text{Noise}, \text{Cost})$?
- How can complex utility functions be assessed from preference behaviour?
 - Idea 1: identify conditions under which decisions can be made without complete identification of $U(x_1, \dots, x_n)$
 - Idea 2: identify various types of **independence** in preferences and derive consequent canonical forms for $U(x_1, \dots, x_n)$

- Typically define attributes such that U is **monotonic** in each
- **Strict dominance**: choice B strictly dominates choice A iff $\forall i X_i(B) \geq X_i(A)$ (and hence $U(B) \geq U(A)$)



- Strict dominance seldom holds in practice

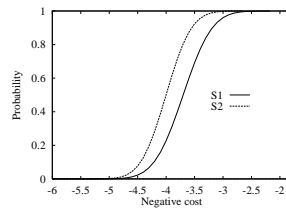
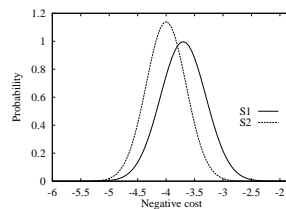
- Distribution p_1 **stochastically dominates** distribution p_2 iff

$$\forall t \int_{-\infty}^t p_1(x) dx \leq \int_{-\infty}^t p_2(x) dx$$

- If U is monotonic in x , then A_1 with outcome distribution p_1 stochastically dominates A_2 with outcome distribution p_2 :

$$\int_{-\infty}^{\infty} p_1(x)U(x)dx \geq \int_{-\infty}^{\infty} p_2(x)U(x)dx$$

- Multiattribute case: stochastic dominance on all attributes \Rightarrow optimal



- Stochastic dominance can often be determined without exact distributions using **qualitative** reasoning
- E.g., construction cost increases with distance from city
 - S_2 is further from the city than S_1
 - $\Rightarrow S_1$ stochastically dominates S_2 on cost
- E.g., injury increases with collision speed
- Can annotate belief networks with stochastic dominance information:
 - $X \overset{+}{\rightarrow} Y$ (X positively influences Y) means that
 - For every value \mathbf{z} of Y 's other parents \mathbf{Z}
 - $\forall x_1, x_2 \ x_1 \geq x_2 \Rightarrow \mathbb{P}(Y|x_1, \mathbf{z})$ stochastically dominates $\mathbb{P}(Y|x_2, \mathbf{z})$

PREFERENCE STRUCTURE: DETERMINISTIC

- X_1 and X_2 **preferentially independent** of X_3 iff preference between $\langle x_1, x_2, x_3 \rangle$ and $\langle x'_1, x'_2, x_3 \rangle$ does not depend on x_3
- E.g., $\langle \text{Noise}, \text{Cost}, \text{Safety} \rangle$:
 $\langle 20,000 \text{ suffer}, \$4.6 \text{ billion}, 0.06 \text{ deaths/mpm} \rangle$ vs.
 $\langle 70,000 \text{ suffer}, \$4.2 \text{ billion}, 0.06 \text{ deaths/mpm} \rangle$
- **Theorem** (Leontief, 1947): if every pair of attributes is P.I. of its complement, then every subset of attributes is P.I. of its complement: **mutual P.I.**
- **Theorem** (Debreu, 1960): mutual P.I. $\Rightarrow \exists$ **additive** value function:

$$V(S) = \sum_i V_i(X_i(S))$$

$$V(\text{noise}, \text{cost}, \text{death}) = -\text{noise} \times 10^4 - \text{cost} - \text{deaths} \times 10^{12}$$

- Hence assess n single-attribute functions; often a good approximation

PREFERENCE STRUCTURE: STOCHASTIC

- Need to consider preferences over lotteries:
X is **utility-independent** of **Y** iff preferences over lotteries **X** do not depend on **Y**
- Mutual U.I.: each subset is U.I. of its complement
 $\Rightarrow \exists$ **multiplicative** utility function:
$$U = k_1 U_1 + k_2 U_2 + k_3 U_3$$
$$+ k_1 k_2 U_1 U_2 + k_2 k_3 U_2 U_3 + k_3 k_1 U_3 U_1$$
$$+ k_1 k_2 k_3 U_1 U_2 U_3$$
- Routine procedures and software packages for generating preference tests to identify various canonical families of utility functions

VALUE OF INFORMATION: SIMPLE EXAMPLE

- One of the most important part of decision making: **know what questions to ask**
- Idea: compute value of acquiring each possible piece of evidence Can be done **directly from decision network**
- Example: buying oil drilling rights
Two blocks A and B , exactly one has oil, worth k
Prior probabilities 0.5 each, mutually exclusive
Current price of each block is $k/2$
Consultant offers accurate survey of A . Fair price?
- Solution: compute expected value of information
= expected value of best action given the information
minus expected value of best action without information
Survey may say "oil in A " or "no oil in A ", prob. 0.5 each
= $[0.5 \times \text{value of "buy A" given "oil in A"}$
+ $0.5 \times \text{value of "buy B" given "no oil in A"}] - 0$
= $(0.5 \times k/2) + (0.5 \times k/2) - 0 = k/2$

VALUE OF INFORMATION: GENERAL FORMULA

- Current evidence E , current best action α
Possible action outcomes S_i , potential new evidence E_j

$$EU(\alpha|E) = \max_a \sum_i U(S_i) P(S_i|E, a)$$

Suppose we knew $E_j = e_{jk}$, then we would choose $\alpha_{e_{jk}}$ s.t.

$$EU(\alpha_{e_{jk}}|E, E_j = e_{jk}) = \max_a \sum_i U(S_i) P(S_i|E, a, E_j = e_{jk})$$

E_j is a random variable whose value is *currently* unknown
 \Rightarrow must compute expected gain over all possible values:

$$VPI_E(E_j) = \left(\sum_k P(E_j = e_{jk}|E) EU(\alpha_{e_{jk}}|E, E_j = e_{jk}) \right) - EU(\alpha|E)$$

(VPI = value of perfect information)

- **Nonnegative**—in *expectation*, not *post hoc*

$$\forall j, E \quad VPI_E(E_j) \geq 0$$

- **Nonadditive**—consider, e.g., obtaining E_j twice

$$VPI_E(E_j, E_k) \neq VPI_E(E_j) + VPI_E(E_k)$$

- **Order-independent**

$$VPI_E(E_j, E_k) = VPI_E(E_j) + VPI_{E, E_j}(E_k) = VPI_E(E_k) + VPI_{E, E_k}(E_j)$$

- Note: when more than one piece of evidence can be gathered, maximizing VPI for each to select one is not always optimal
 ⇒ evidence-gathering becomes a **sequential** decision problem

- a) Choice is obvious, information worth little
- b) Choice is nonobvious, information worth a lot
- c) Choice is nonobvious, information worth little

