State Transition Diagrams

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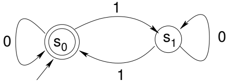
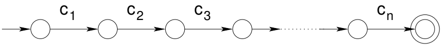
State Transition Diagrams

- Finite directed graph
- Edges (transitions) labeled with symbols from an alphabet
- Nodes (states) labeled only for convenience
- One initial state
- Several accepting states

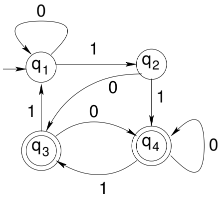


- A string $c_1 c_2 c_3 \ldots c_n$ is accepted by a state transition diagram if there exists a path from the starting state to an accepting state such that the sequence of labels along the path is $c_1, c_2, \ldots, c_n$



  - Same state might be visited more than once
  - Intermediate states might be accepting (but it does not matter)
- The set of exactly all the strings accepted by a state transition diagram is the language accepted (or recognized) by the state transition diagram

- A state diagram describes graphically a deterministic finite automaton (DFA), a machine that at any given time is in one of finitely many states, and whose state changes according to a predetermined way in response to a sequence of input symbols
- Formal definition: a deterministic finite automaton is a tuple $M = (K, \Sigma, \delta, s, F)$
  - $K \Rightarrow$ finite set of states
  - $\Sigma \Rightarrow$ input alphabet
  - $F \subseteq K \Rightarrow$ set of accepting states
  - $s \in K \Rightarrow$ initial state
  - $\delta : K \times \Sigma \to K \Rightarrow$ transition function



$$K = \{q_1, q_2, q_3, q_4\} \quad \Sigma = \{0, 1\} \quad F = \{q_3, q_4\} \quad s = q_1$$

$$\delta(q_1, 0) = q_1 \quad \delta(q_1, 1) = q_2$$
$$\delta(q_2, 0) = q_3 \quad \delta(q_2, 1) = q_4$$
$$\delta(q_3, 0) = q_4 \quad \delta(q_3, 1) = q_1$$
$$\delta(q_4, 0) = q_4 \quad \delta(q_4, 1) = q_3$$

Software Realization

- Big practical advantages of DFA: very easy to implement:
  - Interface to define a vocabulary and a function to obtain the input tokens

```
typename vocab;        /* alphabet + end-of-string */
const vocab EOS;       /* end-of-string pseudo-token */
vocab gettoken(void); /* returns next token */
```

  - Variable (state) changed by a simple `switch` statement as we go along

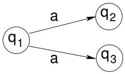
```
int main (void) {
    typedef enum {S0, S1, ... } state;
    state s = S0;    vocab t = gettoken();
    while ( t != EOS ) {
        switch (s) {
            case S0: if (t == ...) s = ...; break;
                     if (t == ...) s = ...; break;
                     ...
            case S1: ...
            ...
        } /* switch */
        t = gettoken();   } /* while */
    /* accept iff the current state s is final */
}
```

Software Realization: Example

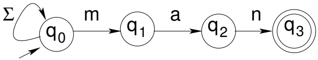
```
typedef enum {ZERO, ONE, EOS} vocab;

vocab gettoken(void) {
    int c = getc(stdin);
    if (c == '0')  return ZERO;
    if (c == '1')  return ONE;
    if (c == '\n') return EOS;
    perror("illegal character");    }

int main (void) {
    typedef enum {S0, S1 } state;
    state s = S0;    vocab t = gettoken();
    while ( t != EOS ) {
        switch (s) {
            case S0: if (t == ONE)  s = S1; break;
                  /* if (t == ZERO) s = S0; break */
            case S1: if (t == ONE)  s = S0; break;
                  /* if (t == ZERO) s = S1; break */   } /* switch */
        t = gettoken(); } /* while */
    if (s != S0) printf("String not accepted.\n");     }
```

Nondeterminism

- So far the state diagrams are deterministic = for any pair (state, input symbol) there can be at most one outgoing transition
- A nondeterministic diagram allows for the following situation:



- The acceptance condition remains unchanged:
- Why nondeterminism?
  - Simplifies the construction of the diagram



  - A nondeterministic diagram can be much smaller than the smallest possible deterministic state diagram that recognizes the same language
- Also known as nondeterministic finite automata (NFA)

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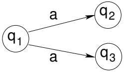

- The acceptance condition remains unchanged:
  - A string $c_1c_2c_3 \ldots c_n$ is accepted by a state transition diagram if there exists some path from the starting state to an accepting state such that the sequence of labels along the path is $c_1, c_2, \ldots, c_n$
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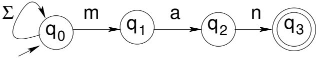


  - A nondeterministic diagram can be much smaller than the smallest possible deterministic state diagram that recognizes the same language
- Also known as nondeterministic finite automata (NFA)

Software Realization

- As above, except that we have to keep track of a set of states at any given time

```
typedef enum { Q0, Q1, Q2, Q3 } state;

int main (void) {
    vocab t = gettoken(); StateSet A;  A.include(Q0);
    while (t != EOS) {
        StateSet NewA;
        for (state s in A) {
            switch (s) {
              case Q0: NewA.include(Q0);
                       if (t == 'm') NewA.include(Q1); break;
              case Q1: if (t == 'a') NewA.include(Q2); break;
              case Q2: if (t == 'n') NewA.include(Q3); break;
              case Q3: break;
            }
        }
        A = NewA;  t = gettoken();
    }
    /* accept iff (Q3 in A) */
}
```

Software Realization (cont'd)

- This kind of implementation is fine for "throw-away" automata
  - Text editor search function searches for a pattern in the text
  - The next search is likely to be different so a brand new automaton needs to be created
- Some times the automaton is created once and then used multiple times
  - The lexical structure of a programming language is well established
  - Lexical analysis in a compiler is accomplished by an automaton that never changes
  - In such a case it is more efficient to precalculate the set of states
    - Exactly as in the previous program
    - Except that we no longer have an input to guide us, so we calculate the sets `NewA` for all possible inputs
    - We obtain a DFA that is equivalent to the given NFA (i.e., recognizes the same language)

- Precalculating all the sets of states effectively constructs a deterministic state transition diagram that is equivalent to the original (nondeterministic) state transition diagram:

algorithm DETERMINIZE($M = (K, \Sigma, \Delta, s, F)$) **returns** $M' = (K', \Sigma, \delta', s', F')$**:**

- $S \leftarrow \{\{s\}\}$ (active states)
- $K' \leftarrow \emptyset$ (done states)
- $\delta' \leftarrow \emptyset$ (start with no transitions)
- **while** $S \neq \emptyset$ **do**
  - Choose $A \in S$ (any state will do)
  - $S \leftarrow S \setminus \{A\}$
  - $K' \leftarrow K' \cup \{A\}$ (state A processed now)
  - **foreach** $a \in \Sigma$ **do** (each action will lead to a new state *NewA*)
    - $NewA \leftarrow \emptyset$
    - **foreach** $(p, a, q) \in \Delta \wedge p \in A$ **do**
      - $NewA \leftarrow NewA + q$ (for every p in A and $p \xrightarrow{a} q$ we add q)
    - **if** $NewA \neq \emptyset$ **then** (if *NewA* is empty then there is no transition)
      - Add to δ' transition $A \xrightarrow{a} NewA$
      - **if** $NewA \notin S \cup K'$ **then** if *NewA* is processed we are done
        - $S \leftarrow S \cup \{NewA\}$ (otherwise we add it to the queue)

PRECALCULATING STATE SETS

- Precalculating all the sets of states effectively constructs a deterministic state transition diagram that is equivalent to the original (nondeterministic) state transition diagram:

algorithm DETERMINIZE($M = (K, \Sigma, \Delta, s, F)$) **returns** $M' = (K', \Sigma, \delta', s', F')$**:**

```
S ← {{s}}                                                        (active states)
K' ← ∅                                                             (done states)
δ' ← ∅                                                (start with no transitions)
while S ≠ ∅ do
    Choose A ∈ S                                                (any state will do)
    S ← S \ {A}
    K' ← K' ∪ {A}                                          (state A processed now)
    foreach a ∈ Σ do                      (each action will lead to a new state NewA)
        NewA ← ∅
        foreach (p, a, q) ∈ Δ ∧ p ∈ A do
            NewA ← NewA + q               (for every p in A and p -a-> q we add q)
        if NewA ≠ ∅ then             (if NewA is empty then there is no transition)
            Add to δ' transition A -a-> NewA
            if NewA ∉ S ∪ K' then              if NewA is processed we are done
                S ← S ∪ {NewA}                 (otherwise we add it to the queue)
s' ← {s}
```

- Precalculating all the sets of states effectively constructs a deterministic state transition diagram that is equivalent to the original (nondeterministic) state transition diagram:

```
algorithm DETERMINIZE(M = (K, Σ, Δ, s, F)) returns M′ = (K′, Σ, δ′, s′, F′):
    S ← {{s}}                                              (active states)
    K′ ← ∅                                                   (done states)
    δ′ ← ∅                                       (start with no transitions)
    while S ≠ ∅ do
        Choose A ∈ S                                     (any state will do)
        S ← S \ {A}
        K′ ← K′ ∪ {A}                                (state A processed now)
        foreach a ∈ Σ do          (each action will lead to a new state NewA)
            NewA ← ∅
            foreach (p, a, q) ∈ Δ ∧ p ∈ A do
                NewA ← NewA + q       (for every p in A and p -a-> q we add q)
            if NewA ≠ ∅ then     (if NewA is empty then there is no transition)
                Add to δ′ transition A -a-> NewA
                if NewA ∉ S ∪ K′ then      if NewA is processed we are done
                    S ← S ∪ {NewA}      (otherwise we add it to the queue)
    s′ ← {s}
    F′ ← {p ∈ K′ : K′ ∩ F ≠ ∅}               (a single accepting state will do)
```

ε-TRANSITIONS

- Useful at times to have “spontaneous” transitions = transitions that change the state without any input being read = ε-transitions
  - Only available for nondeterministic state transition diagrams!
- Example of usefulness: Construct the state transition diagram for the language

$$\{0,1\}^*01\{0,1\}^* + \{w \in \{0,1\}^* : w \text{ has an even number of 1's}\}$$

- Even better ε-transitions can be eliminated afterward

Eliminating ε-Transitions

For every diagram M with ε-transitions a new diagram without ε-transitions can be constructed as follows:

1. Make a copy M' of M where the ε-transitions have been removed. Remove states that have only ε-transitions coming in except for the starting state
2. Add transitions to M' as follows: whenever M has a chain of ε-transitions followed by a “real” transition on x:

   $$\textcircled{q} \xrightarrow{\varepsilon} \bigcirc \xrightarrow{\varepsilon} \cdots \xrightarrow{\varepsilon} \bigcirc \xrightarrow{x} \textcircled{p}$$

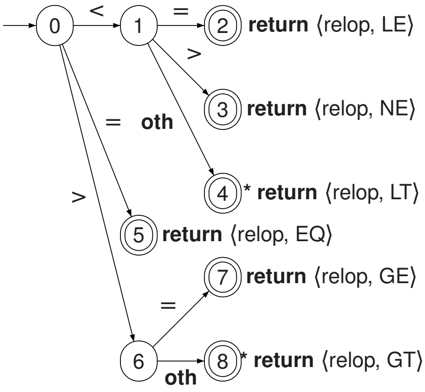
   add to M' a transition from state q to state p labeled by x:

   $$\textcircled{q} \xrightarrow{x} \textcircled{p}$$

   - Note that q and p may be any states
   - In particular this step is also used in the case where $q = p$
3. If M has a chain of ε-transitions from a state r to an accepting state, then r is made to be an accepting state of M'.

EXAMPLES FROM LEXICAL ANALYSIS

- Lexical analysis splits the input of a compiler (program) into lexical units (tokens)
- First step of compilation, easy to implement using state transition diagrams



When returning from *-ed states must "put back" the last character read