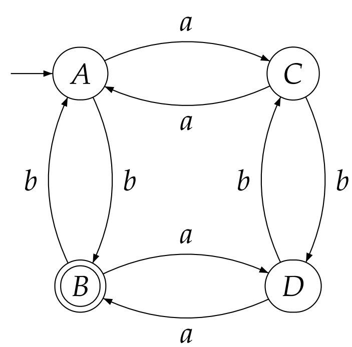
CS 310, Assignment 2

Answers

1. Let $\Sigma = \{a, b\}$ and consider the following state-transition diagram:



   (a) Give three examples of strings that are accepted by the transition diagram and three examples of strings that are not accepted by the transition diagram.

   ANSWER: Strings that are accepted by the automaton include: *b*, *bbb*, *aba*, *abbba*, *aaab*. Strings that are not accepted include: *a*, *aa*, *ab*, *abb*, *abbb*.

   (b) Write explicitly the transition function δ that defines the transitions of the diagram.

   ANSWER:

$\delta(A, a) = C$	$\delta(A, b) = B$	$\delta(C, a) = A$	$\delta(C, b) = D$
$\delta(B, a) = D$	$\delta(B, b) = A$	$\delta(D, a) = B$	$\delta(D, b) = C$

   (c) Is the transition diagram deterministic or nondeterministic? Explain briefly.

   ANSWER: The automaton is deterministic as there are no ε-transitions and each state has only one transition enabled for every input symbol.

   (d) What is the language recognized by the state transition diagram? Describe (in English) conditions that characterize exactly all the strings in the language.

ANSWER: The language contains exactly all the binary strings that have an even number of a symbols and an odd number of b symbols.

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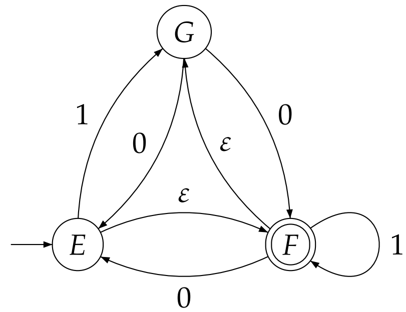
2. Let $\Sigma = \{0, 1\}$ and consider the following nondeterministic state transition diagram with ε-transitions:

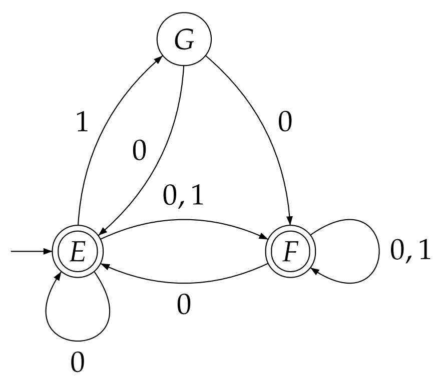


Using the systematic method described in class (and in the text), convert the transition diagram into an equivalent (non)deterministic transition diagram without ε-transitions. Do not modify or simplify the resulting diagram any further.

ANSWER: The following paths will need to be replaced by single transitions:

$E \xrightarrow{\varepsilon} F \xrightarrow{1} F$	becomes	$E \xrightarrow{1} F$	$E \xrightarrow{\varepsilon} F \xrightarrow{0} E$	becomes	$E \xrightarrow{0} E$
$F \xrightarrow{\varepsilon} G \xrightarrow{0} F$	becomes	$F \xrightarrow{0} F$	$F \xrightarrow{\varepsilon} G \xrightarrow{0} E$	becomes	$F \xrightarrow{0} E$
$E \xrightarrow{\varepsilon} F \xrightarrow{\varepsilon} G \xrightarrow{0} E$	becomes	$E \xrightarrow{0} E$	$E \xrightarrow{\varepsilon} F \xrightarrow{\varepsilon} G \xrightarrow{0} F$	becomes	$E \xrightarrow{0} F$

The two ε-transitions $E \xrightarrow{\varepsilon} F$ and $F \xrightarrow{\varepsilon} G$ are then removed. State E becomes accepting since there is an ε-path from it to an accepting state. We end up with the following:



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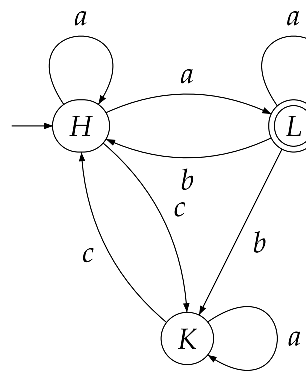
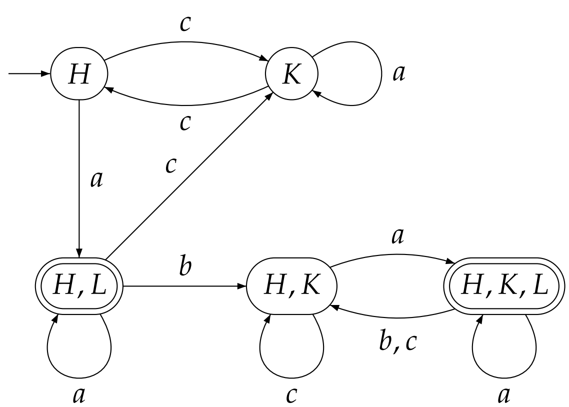
3. Let $\Sigma = \{a, b, c\}$. Using the systematic method described in class and textbook convert the following nondeterministic state transition diagram into a deterministic transition diagram:



Describe how the deterministic transition diagram is obtained from the nondeterministic one as follows: label the states of the deterministic diagram by sets of states of the nondeterministic diagram (like we did in class).

ANSWER:



The only accepting state in the original automaton is L and so the accepting states in the deterministic automaton are the two that contain L namely, $\{H, L\}$ and $\{H, K, L\}$.

For people needing yet another example here is a walk-through:

(a) We start from the initial state H (technically $\{H\}$ except that we do not put braces around singletons). With an a we can stay in H or go to L (so we end up in $\{H, L\}$); with a c we can reach only K; we do not go anywhere with a b. The new states that need to be considered are K and $\{H, L\}$.

(b) From K: we stay in K with an a, we can only reach H with a c, and we do not go anywhere with a b. The state H has already been considered so the only new state left is $\{H, L\}$ (from the previous point).

(c) From $\{H, L\}$: H goes into either H or L with an a, while L goes back into L with the same a. Overall $\{H, L\}$ yields $\{H, L, L\} = \{H, L\}$ with an a. Similarly H does not go anywhere with a b, but L can become either H or K with the same b. Overall $\{H, L\}$ becomes $\{H, K\}$ after a b. Finally H becomes K with a c, whereas L does not go anywhere. That is, $\{H, L\}$ becomes K after a c. There is a new state thus introduced namely, $\{H, K\}$.

(d) From $\{H, K\}$: H becomes H or L with an a, while K stays K with the same a. Overall $\{H, K\}$ becomes $\{H, K, L\}$ with that a. H becomes K with a c and the other way around that is, $\{H, K\}$ stays $\{H, K\}$ with a c. We have yet another new state namely, $\{H, K, L\}$.

(e) From $\{H, K, L\}$: After an a H becomes H or L, K remains K, and L remains L for the overall state $\{H, K, L\}$. With a b L becomes H or K while neither H nor K do anything, for the overall state $\{H, K\}$. Finally, when a c is received H becomes K and the other way around, while L does not do anything, once again for an overall state $\{H, K\}$. There are no states left to consider so we are done.
