CS 317, Assignment 1

Due on 24 September in class

1. Consider the following algorithm which receives as input an array A of size n:

```
i ← 1
while i ≤ n do
    sum_i ← 0
    prod_i ← 1
    for j = 1 to n do
        sum_i ← sum_i + A_j
    for j = 1 to n do
        prod_i ← prod_i + A_j
    i ← i + 1
```

   - State how many times each loop is executed and justify your answer.
   - Give the running time of the algorithm in Θ notation. Explain how you reached the answer.

2. Consider the following algorithm which receives as input two numbers m and n and sets $result$ to $m \times n$; `div` is the integer division operator.

```
result ← 0
repeat
    if m is odd then
        result ← result + n
    m ← m div 2
    n ← n + n
until m < 1:
```

   - State (as a function of m and n) how many times the loop is executed and justify your answer.
   - Give the running time of the algorithm in Θ notation as a function of m and n. Explain how you reached the answer.

3. Prove each of the following statements without using limits. Justify your answer.

   (a) $3n^3 + 2n^2 + n \in \Omega(n^2)$

   (b) $2^n \in \Theta(2^{n-2})$

   (c) $(\log n^2) \in o(\log n)^2$

   (d) $2^{(n+1)} \in O(4^n)$

(e) $2^{2n} \notin \Theta(2^n)$

4. For each relation below find *all* the $\mathbb{X} \in \{O, \Omega, \Theta, o, \omega\}$ that make the relation true. Justify your answer *using limits*.

   (a) $3n^3 + 2n^2 + n \in \mathbb{X}(n^3)$

   (b) $(n \log n)^2 \in \mathbb{X}(n^2 \log n^2)$

   (c) $n^2 + 2^n \in \mathbb{X}(n2^n)$

   (d) $(n-1)! \in \mathbb{X}(n!)$

   (e) $n \log n \in \mathbb{X}(\sqrt{n})$

Make sure you review the submission guidelines posted on the course's Web site before submitting.