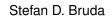


# Introduction to the Design and Analysis of Algorithms



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- It all starts with a problem: a task to be performed or a question to be answered
  - Sort a sequence S of n numbers in increasing order
  - 2 Determine whether the number *x* is in the sequence *S* of *n* numbers
  - Find the *n*th term in the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, ...
- Parameters: variables that are not assigned values in the statement of the problem
  - 🚺 S, n
  - 2 S, n, x
  - 3 n
- Algorithm: A rigorous, step-by-step procedure to solve a problem for all possible values of the parameters
  - Named after Abū 'Abdallāh Muḥammad ibn Mūsā al-Khwārizmī, or Mohammed Al-Khorezmi for short (Baghdad, 780–850)

### PROBLEMS AND ALGORITHMS (CONT'D)

- Instance: A specific assignment of values to the parameters
  - **()** Sorting instance:  $S = \langle 8, 3, 5, 6, 3, 9, 2 \rangle, n = 7$
  - **2** Searching instance:  $S = \langle 8, 3, 5, 6, 3, 9, 2 \rangle$ , n = 7, x = 9
  - Sibonacci calculation instance: n = 4
- Solution: The answer to the question posed in the problem on the given instance
  - $S = \langle 2, 3, 3, 5, 6, 8, 9 \rangle$
  - 2 Yes/True
  - 3 🗿
- A traditional algorithm:
  - Receives an input
  - Produces an output
  - Is deterministic i.e., all the intermediate results are unambiguously determined by the previous steps and input
  - It is correct (aka partial correctness)
  - It always terminates (aka total correctness)
  - It is general in the sense that it works for any set of input values

## DESIGN AND ANALYSIS OF ALGORITHMS

- Algorithm design (the Art)
  - Can consider different techniques such as
    - Divide and conquer

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- Greedy
- Dynamic programming
- Backtracking
- Branch and bound
- Algorithm analysis (the Science)
  - Proof of partial and total correctness
  - Performance analysis (time and space)
- Throughout the course we will describe algorithms using pseudocode
  - Flexible enough to allow for concise descriptions, but rigorous enough to be easily translated into actual code in any half-decent programming language

### **ALGORITHM DESIGN**



There are multiple algorithms for most problems

٩	Find the largest of four val-
	ues <i>a</i> , <i>b</i> , <i>c</i> , <i>d</i>

- Which algorithm is More time efficient? (number of comparisons!)
  - More space efficient?
  - More elegant? (e.g., simpler)

largest  $\leftarrow$  a if  $\tilde{b} > largest$  then | largest  $\leftarrow$  b if c > largest then largest  $\leftarrow c$ if d > largest then largest  $\leftarrow d$ return largest

#### else return d else if c > d then **return** c else return d else if b > c then if b > d then return b else return d else if c > d then return c else return d

if a > b then

if a > c then

if a > d then

return a

Multiplication of two integers

$\frac{\text{Traditional}}{(0981 \times 0123)}$	Divide and conquer $(09 81 \times 01 23)$
981	09 81
x 123	x 01 23
2943	18 63 (23 x 81)
1962	207 (23 x 09)
981	81 (01 x 81)
	09 (01 x 09)
120663	
	1206 63

Peasant multiplication  $(m \times n)$ 

### *result* $\leftarrow$ 0 repeat if *m* is odd then l result $\leftarrow$ result + n $m \leftarrow m \operatorname{div} 2$ $n \leftarrow n + n$

**until** *m* < 1:

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ALGORITHM DESIGN (CONT'D)

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## ALGORITHM DESIGN (CONT'D)

Sequential search:

• Searching for a given value in a sequence of values

- Computing the *n*th Fibonacci number • Recursive: algorithm FIB (*n*): if n < 1 then return n else **return** FIB (*n*−1) + FIB (*n*−2)
  - Iterative:

algorithm FIB (n):  $f[0] \leftarrow 0$ if n > 0 then  $f[1] \leftarrow 1$ for i = 2 to n do  $| f[i] \leftarrow f[i-1] + f[i-2]$ return f[n]

- Which algorithm is more elegant?
- Which algorithm is faster?

```
algorithm SEQSEARCH(x, S, I, h):
                 i \leftarrow I
                 while i < h do
                      if S[i] = x then return i
                      else i \leftarrow i + 1
                return -1
          Binary search:
             algorithm BINSEARCH(x, S, I, h):
                 i ← I
                 j \leftarrow h
                 while i \leq j do
                      m \leftarrow (i+j)/2
                      if S[m] = x then return m
                      else if S[m] > x then j \leftarrow m - 1
                      else i \leftarrow m+1
                return -1
          Speed? Restrictions?
             (BINSEARCH is not an algorithm unless preconditions are stated)
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```



# • The performance of an algorithm (running time, space requirements) must be a function of input size

- Critical to define a meaningful input size
  - Running time may vary widely when different concepts of size are considered
- Example: The running time of an algorithm for multiplying two  $n \times n$  matrices
  - Compare the running time as a function of *n* (the dimension of the matrix) vs. a function of  $n \times n$  (the number of values in one matrix) vs. a function of  $2 \times n \times n$  (the total number of values involved)
  - What would be a fair notion of input size?
- Example: Consider an algorithm that determines whether the input *N* is a prime number
  - Compare the running time as a function of *N* (the input number itself) vs. a function of the number of digits of *N*
  - What would be a fair notion of input size?

### TIME COMPLEXITY

- During the course we will mostly analyze algorithms with respect to their running time
  - Arguably the most significant measure of performance
- Running time can vary depending on many other factors than the size of the input, such as the power of the machine the implementation will run on
  - We want a measure of performance that is independent of such factors
  - We will split the running time of algorithms into classes that ignore this kind of factors (multiplicative or additive constants) ⇒ time complexity
  - We will further analyze the time complexity of an algorithm as the input size keeps increasing indefinitely ⇒ asymptotic time complexity
- The running time of many algorithms depends of the particular instance the algorithm runs on, so one may consider
  - worst-case time complexity (used the most often)
  - average-case time complexity (used sometimes)
  - best-case time complexity (not very meaningful, rarely used if ever)
- Amortized complexity determines the running time an algorithm is statistically likely to need (under various definitions of "likely")
  - Most useful for operations over data structures and also online algorithms

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