CS 467/567, Assignment 3

Due on 24 March

Here is a little assignment on computational geometry. The submission is tighter than usual because the problems are simpler than usual as well.

This assignment is individual. Submit your solutions by email as a single document typeset to PDF. I recommend that you solve the problems by yourself with no external references, but if references are used then they must be provided and also cited in the text.

Problem 1: Segment intersection

This question refers to the ANY-SEGMENTS-INTERSECT algorithm as described in the lectures as well as Cormen et. al. Does this algorithm work correctly even if three or more segments intersect at the same point? Prove one way or another.

Problem 2: Ghostbusters and ghosts

A group of n Ghostbusters is battling n ghosts. Each Ghostbuster carries a proton pack, which shoots a stream at a ghost, eradicating it. A stream goes in a straight line and terminates when it hits the ghost. The Ghostbusters decide upon the following strategy: They will pair off with the ghosts, forming n Ghostbuster-ghost pairs, and then simultaneously each Ghostbuster will shoot a stream at his chosen ghost. As we all know, it is *very* dangerous to let streams cross, and so the Ghostbusters must choose pairings for which no streams will cross.

Assume that the position of each Ghostbuster and each ghost is a fixed point in the plane and that no three positions are colinear.

1. Prove that there exists a line passing through one Ghostbuster and one ghost such that the number of Ghostbusters on one side of the line equals the number of ghosts on the same side. Design an algorithm that finds such a line in $O(n \log n)$ time.

2. Give an $O(n^2 \log n)$-time algorithm to pair Ghostbusters with ghosts in such a way that no streams cross.