**Assignment1: Partition Problem**

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**Partition Problem Definition:**

Given a set of nonnegative integers (A), we want to know is there a subset $S \subseteq A$ which sum of all the values in S will be equal to sum of all the values not in S.

$\sum S = \sum (A - S) = (\sum A) / 2$

**Proof of partition membership in NP:** [1][2]

**PARTITION ∈ NP**

In this assignment we consider A a set of positive integers, and S1 and S2 are two subsets (partitions) of A that we want to compare them to see whether they are equal.

Here is a **polynomial-time** verifier for PARTITION:

● If $S1 \cup S2 = A$ and that $S1 \cap S2 = \emptyset$.

● Then we compare the sum of values in S1 with sum of the elements in S2

● If above sums are equal we accept.

● If above sums are not equal we do not accept.

If we want to make simple example for it, assume that A= {3,4,6,5} S1= {3,6} S2= {4,5}

Then we understand $S1 \cup S2 = A$ and $S1 \cap S2 = \emptyset$. Also S1=S2 So we accept it.

**PARTITION ∈ NPC** [2][3]

Reduction from Subset Sum:

We assume A is a set of nonnegative integers and the target value is k. S is a subset of A, and the sum of members of S, equals k. So (A, k) ∈ Subset sum. Sum of all integers in A equals m.

$A = \{a_1, \ldots., a_n\}$

$\sum A = m$

$\sum S = k$

We create a new set $A' = A \cup \{a_{n+1}, a_{n+2}\}$

Such that: $a_{n+1} = 2m - k$ and $a_{n+2} = m + k$

This can be done in polynomial time.

$\sum A' = m + (2m - k) + (m + k) = 4m$

We also create the partitions of A' which are S' and (A'-S')

$S' = S \cup a_{n+1}$ and $(A' - S') = (A - S) \cup a_{n+2}$

So, we have:

$\sum S' = k + (2m - k) = 2m$

$\sum (A' - S') = (m - k) + (m + K) = 2m$

So, now we have a set of integers A' that has a subset S', such that

$\sum S' = \sum (A'-S') = (\sum A') / 2$

And therefore, A' is an instance of Partition.

Also, if we assume A' is an instance of Partition, which sum of each partition equals 2m, then one of the partitions has the member $a_{n+1}$ with value (2m – k). Sum of $a_{n+1}$ and $a_{n+2}$ is 3m, and each partition should have the sum 2m. so $a_{n+1}$ and $a_{n+2}$ cannot be in the same partition. So, if we remove $a_{n+1}$ from this subset, the sum of members in the subset would be 2m – (2m – k) = k. and if we remove an+2 from the other subset, we get A, which has a subset with sum of k, therefore (A, k) is true instance of Subset sum.

(A') ∈ Partition iff (A, k) ∈ Subset Sum

So, we reduced from subset sum to partition; And because we know subset sum is NP complete, therefore, partition is NP Complete too.

**Optimization of partition problem:**[5]

Partition decision problem, is deciding if a set has a subset, which sum of all of its values equals to sum of items not in the subset. Optimization version of the partition problem suggests partition of the set A into two subsets S1, S2 which the differences between the sum of each subset is minimized (as nearly equal as possible).

**References:**


[1] https://web.stanford.edu/class/archive/cs/cs103/cs103.1132/lectures/27/Small27.pdf

[2] https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-046j-design-and-analysis-of-algorithms-spring-2015/lecture-videos/lecture-16-complexity-p-np-np-completeness-reductions/

[3] https://www.cs.mcgill.ca/~jmerce1/a4ans.html

[4] https://www.geeksforgeeks.org/pseudo-polynomial-in-algorithms/

[5] Multi-Way Number Partitioning, Richard E. Korf Computer Science Department University of California, Los Angeles Los Angeles, CA