

CS 467/567, Assignment 1

Due on 11 February

This assignment can be solved in groups consisting of no more than *two* students.

1 The problems

For each of the two problems below provide a complete proof of NP-completeness. Also state (without proof or justification) what is the optimization problem from which each decision problem is derived.

1. SET COVER: A set covering instance is a pair (U, S) for a finite set U and $S \subseteq 2^U$ such that $U = \bigcup_{X \in S} X$. We say that S covers U . Given a set covering instance (U, S) and an integer k , is there a subset $C \subseteq S$, $|C| = k$, whose members cover U .
2. KNAPSACK: Given n objects with volumes v_i and prices s_i , $1 \leq i \leq n$, a knapsack of volume V , and a constant $S > 0$, is there a subset $N' \subseteq \{1, \dots, n\}$ such that $\sum_{i \in N'} v_i < V$ (the objects in N' can be placed in the knapsack) and $\sum_{i \in N'} s_i \geq S$ (their price exceeds the given limit S)?

Trivial variants (such as the size of the solution being equal rather than smaller or larger than the given constant or the other way around) are acceptable.

You are required to present your proofs starting from one (or more) of the problems established as being NP-complete in class (Bounded tiling, SAT, CLIQUE, VERTEX COVER, and HAMILTONIAN CYCLE). If you need in the process to consider other problems, then you must provide a proof for the full chain by proving that all the intermediate problems are NP-complete.

The good news is that the two problems are nothing new; in fact they are between the oldest problems known to be NP-complete. As such your proofs need not be original. Instead you are asked to find the existing proofs of NP-completeness and present them in a coherent report. You *must* list all your references and you *must* cite your references in the text as appropriate.

This is being said, your proofs must still be formal and complete; if they are not completely presented in your references then you must complete them. They must also be stated in your own words. Copying and pasting from your references is *not* acceptable; instead you *must* understand the matter presented in your references and write down the respective proofs in your own words.

2 Deliverable

Hand in a report containing your proofs. The report must present formal and complete proofs. Cite all your sources. In particular the report should cite at least one technical reference (journal paper, conference paper, technical report, or textbook). Encyclopedic reference (such as to Wikipedia articles) and references to course lecture notes are not acceptable.

Your report will be marked for completeness (no pun intended), originality (in the sense explained above), clarity, and references (plus their citation in the text). The report must be typeset to PDF and must be handed in by email.