The Independent Set and NP-Completeness

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Proof that INDEPENDENT SET is NP-complete

We begin by having an undirected simple graph, G = (V, E), letting a subset I ⊆ V in G be an independent set if no two vertices in set I are connected by an edge in G.[1] For every vertex in I, we must check that there is no edge of G that connects this vertex to *at leas*t another vertex in I.[1] If we run into such an edge, we reject such a set I, otherwise it is accepted as an independent set of the graph G.[1] This means the algorithm runs in polynomial time, so IS is in NP.[1]

Proof of INDEPENDENT SET is NP-hardness

It suffices to show that some known NP-complete problem (3SAT) is polynomially reducible to IS, that is, 3SAT $\leq_P$ IS.[2] Let F be a boolean formula in 3-CNF form, we are looking to find a polynomial time computable function we shall call f, for which that F, the boolean formula, into an input for the IS problem.[2] This function f with an input of the boolean formula F shall output a graph, G, and an integer k, which is shown below in the schematic Fig. 1.[2] That is, we are looking for a f(F) = (G, k) with such that F is satisfiable if and only if G has an independent size of k.[2]

This F will imply that if we solve for the independent set for G and k in polynomial time, therefore we should be able to solve 3SAT in polynomial time.[2]

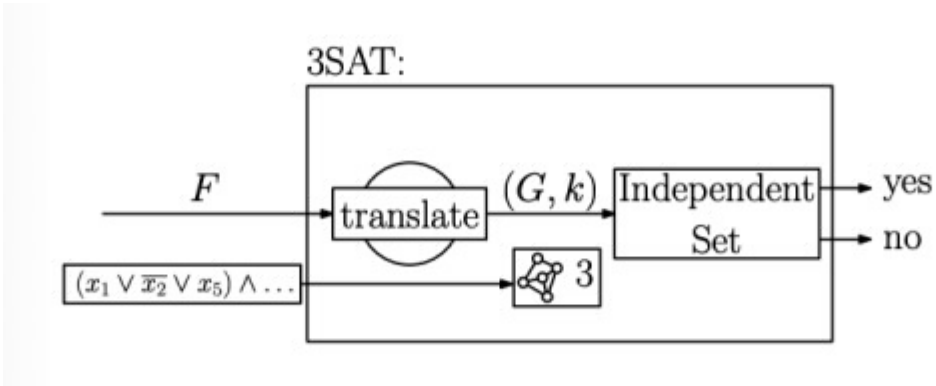

Fig. 1: Reduction of 3-SAT to IS.

Optimization Problem

"Many problems of interest are optimization problems, in which each feasible (i.e., "legal") solution has an associated value, and we wish to find a feasible solution with the best value." [4] In this section we will give a brief overview of some continuous approaches to the independent set problem.[3] As the maximum independent set problem has many equivalent formulations as an integer programming problem, and as a continuous nonconvex optimization problem (Pardalos and Xue, 1992; Bomze et al., 1999).[3]

Motzkin and Straus (1965) had formulated the max clique problem as a standard quadratic programming problem, with its original proof done through induction.[3] Another proof can be seen by Abello et al.[3] (2001). Let AG be the adjacency matrix of G, and allow e to be the n-dimensional vector with all components equal to 1.[3] The optimal value of the following quadratic program is seen below (Theorem 1 Motzkin-Straus).[3]

$$\max f(x) = \frac{1}{2}x^T A_G x,$$

subject to

$$e^T x = 1,$$
$$x \geq 0.$$

is given by

$$\frac{1}{2}\left(1 - \frac{1}{\omega(G)}\right),$$

where $\omega(G)$ is the clique number of G.

The result of this quadratic program is extended in Gibbons et al. (1997), which was done by providing characterization of max cliques in terms of local solutions.[3] Optimality conditions of the MotzkinStraus program have been studied.[3] In addition the properties of newly introduced parametrization of the corresponding quadratic programming, as seen in the figure, have also been investigated.[3]

References Page

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