Bishop's University

Math 106: Final Exam
Fall 2012

Name: _____

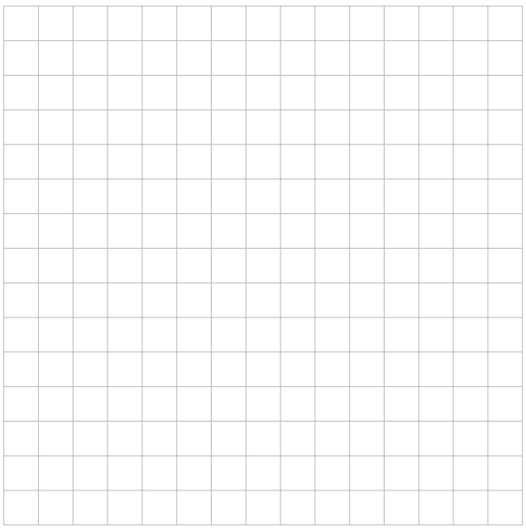
Student #: _____

- This exam is 180 minutes in length.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

Page	Points	Score
2	17	
3	17	
4	9	
5	9	
6	12	
7	6	
8	10	
9	10	
Total:	90	

Part A: Do all of these thirteen (13) questions.

1. (4 points) Find and sketch the domain of the function $f(x,y) = \sqrt{4-x^2-y^2} + \sqrt{1-x^2}$.



2. (3 points) Show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}.$$

3. (6 points) Find the all second partial derivatives of $f(x,y) = 4x^3 - xy^2$.

4. (4 points) Find the equation of the tangent plane to $z = e^x \cos y$ at $(0,0,1)$, and write it in general form.

5. (5 points) If $v = x^2 \sin y + ye^{xy}$, where $x = s + 2t$ and $y = st$, use the Chain Rule to find $\partial v/\partial t$ when $s = 0$ and $t = 1$.

6. (4 points) If $\cos(xyz) = 1 + x^2y^2 + z^2$, find $\partial x/\partial z$.

7. (3 points) Find the direction in which $f(x, y, z) = ze^{xy}$ increases most rapidly at the point $(0, 1, 2)$. What is the maximum rate of increase?

8. (5 points) The two legs of a right angle triangle are measured as 5 m and 12 m, with a possible error in measurement of at most 2 cm. Use differential to estimate the maximum error in the calculated value of the area of the triangle. (**Note: neither of these legs are the hypotenuse.**)

9. (9 points) Classify the critical points of the function $f(x, y) = x^3 - 6xy + 8y^2$.

10. (9 points) Use Lagrange Multipliers to find the maximum and minimum values of $f(x, y) = \dfrac{1}{x} + \dfrac{1}{y}$ subject to the constraint $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$.

11. (6 points) Rewrite the iterated integral $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2) dy dx$ using polar coordinates. **Do NOT evaluate the integral**.

12. (6 points) Rewrite the iterate integral $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2+y^2+z^2} dz dx dy$ using spherical coordinates. **Do NOT evaluate the integral**.

13. (6 points) Use the transformation $u = x-y$, $v = x+y$ to write $\iint_R \frac{x-y}{x+y} dA$ as an interated integral in the order $dudv$, where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$, and $(1,3)$. **Do NOT evaluate the integral**.

Part B:
Do any two (2) questions from Questions 14, 15, and 16.

14. (a) (4 points) Find two unit vectors that are orthogonal to both $\vec{j} + 2\vec{k}$ and $\vec{i} - 2\vec{j} + 3\vec{k}$.

    (b) (2 points) Find the parametric equations for the line through $(4, -1, 2)$ and $(1, 1, 5)$.

    (c) (4 points) Find the general equation of the plane through $(3, -1, 1)$, $(4, 0, 2)$, and $(6, 3, 1)$.

15. (10 points) Find the length of the curve $\vec{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle$, $0 \leq t \leq 1$, and find the curvature at $t = 1$.

16. (10 points)

(a) Calcuate the iterated integral $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$ by first reversing the order of integration.

(b) Calculate the iterated integral $\int_0^1 \int_0^1 ye^{xy} dydx$.