Bishop's University

Math 108: Final Exam
Fall 2017

Name: ______________________

Student #: ______________________

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other electronic calculators are permitted.

Page	Points	Bonus Points	Score
2	20	0	
3	15	0	
4	15	0	
5	15	0	
6	10	0	
8	20	0	
9	20	0	
10	0	5	
Total:	115	5	

1. Consider the following matrices:

$$A = \begin{bmatrix} 5 & 1 \\ -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ -3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (5 points) $3A + 2BC$

(b) (5 points) DBA

(c) (5 points) $3CB + 10I$

(d) (5 points) $((BC)^T)^{-1}$

2. (10 points) For what value of k, if any, will the system have (a) no solution, (b) a unique solution, (c) infinitely many solutions.

$$\begin{aligned} x + ky &= 1 \\ kx + y &= 1 \end{aligned}$$

3. (5 points) Let A be an idempotent matrix, meaning $A^2 = A$. Find all possible values of $\det(A)$.

4. (10 points) Determine if these vectors for a basis for $\mathbb{R}^3$: $\begin{bmatrix}1\\1\\1\end{bmatrix}$, $\begin{bmatrix}1\\2\\3\end{bmatrix}$, and $\begin{bmatrix}1\\3\\6\end{bmatrix}$. If so, write $\begin{bmatrix}1\\-2\\3\end{bmatrix}$ as a linear combination of these vectors.

5. (5 points) Let $S = \left\{ \begin{bmatrix}x_1\\x_2\end{bmatrix} \middle| \, x_2 \text{ is even} \right\}$. Is S a subspace of $\mathbb{R}^2$? Justify your answer.

6. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 2 \end{bmatrix}.$$

(a) (13 points) Find the inverse of A.

(b) (2 points) Solve the following system of equations: $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

7. (10 points) For the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 & 6 \\ 0 & 3 & 5 & 7 & 4 & 6 \\ 0 & 6 & 9 & 12 & 7 & 10 \end{bmatrix}$$

with reduced row echelon form,

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

find a basis for the the column space and the null space of A.

8. Compute the determinant of

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 & 5 \\ 1 & 2 & 3 & 5 & 8 \\ 1 & 3 & 5 & 8 & 13 \\ 1 & 5 & 8 & 13 & 0 \end{bmatrix}$$

9. (a) Find an equation of the plane passing through these three points: $(1, 0, 1), (-1, 2, -3)$, and $(2, 0, -1)$.

(b) Find the line of intersection between the plane $2x+4y+2z = 5$ and the plane $-2x+3y-4z = 24$. (Hint: for a point to be in the intersection, it must be on both planes.)

10. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ and $S : \mathbb{R}^4 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_3 - x_1 \\ x_3 - x_2 \end{bmatrix} \quad \text{and} \quad S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + x_3 - x_4 \\ x_1 + x_2 - x_3 - x_4 \\ -x_1 - x_2 - x_3 + x_4 \end{bmatrix}.$$

(a) (10 points) Find the matrices of linear transformation for S and T.

(b) (5 points) Find the matrix of linear transformation for ST, and give $ST\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$.

(c) (5 points) Is ST invertible? Justify your answer.

11. Let

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

(a) (5 points) Find the eigenvalues of A. (Hint: $\lambda = -2$ is an eigenvalue with multiplicity 2.)

(b) (5 points) Find the eigenvectors of A.

(c) (5 points) Find matrices P and D such that $A = PDP^{-1}$.

(d) (5 points) Use the information above to find A^4.

12. (5 points (bonus)) “Waxwords are a cinch!” said Walt after only two days in the business. He had started small, of course just a one-room shop on a busy street in a drab district. But a flashy sign, lush and sexy and far removed from anything to be seen inside, brought in the people.

He was charging three rates, he said: top price for men, rather lower for women, and a nominal fee for a child. “The first day was fine”, he told Jim, “and I took in $20.10.” Fixing Lady Godiva’s tresses, he went on: “Yesterday, my second day, the same number of people came in but I took a dollar twenty less.”

“How come?” asked Jim, somewhat confused by what he saw.

“Same number of children as there’d been men the first day”, replied Walt, “and of women as thered been kids and of men as there’d been women.” He chuckled happily. “It’s the men I want, and if they’d all been men I’d have taken in thirty dollars yesterday.”

Theres a streak of caution in Jim. “What if they don’t come?” he suggested.

Walt laughed. “It wouldn’t break me,” he boasted. “If they’d all been women I’d have taken fifteen bucks each day, or six bucks even if they’d all been kids.”

Jim could hardly envisage the show as family entertainment, but he asked if there were any reduction in such cases. “Not a cent,” declared Walt. “Just regular rates: eighty-five cents for two parents with a child.”

At that moment a bunch of teenagers entered the place, so Jim went out. But he really would like to know how many men had gone there the first day. Maybe you know?

Taken from *figurets: more fun with figures* by J. A. H. Hunter, published 1958. You may solve this problem using any method learned this term.