Bishop's University

Math 108: Final Exam
Fall 2021

Name: ______________________________

Student #: ______________________________

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	15	
4	15	
5	15	
6	15	
7	10	
8	10	
9	5	
10	15	
Total:	120	

1. Consider the following matrices:

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 1 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (5 points) $3A + 2C^t B^t$

(b) (5 points) $A^2 - 5A + I$

(c) (5 points) $3DB + 10I$

(d) (5 points) $((BC)^t)^{-1}$

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 1 \\ 0 & 2 & -2 \end{bmatrix}.$$

(a) (3 points) Find the determinant of A.

(b) (10 points) Find the inverse of A.

(c) (2 points) Solve the following system of equations: $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

3. (10 points) For what value of k, if any, will the system have (a) no solution, (b) a unique solution, (c) infinitely many solutions.

$$\begin{aligned} 28x + 5ky &= 2 \\ 7kx + 45y &= -3 \end{aligned}$$

4. (5 points) Let A be a 5×5 matrix satisfying $A^2 = -A^{-1}$. Find all possible values of $\det(A)$.

5. (10 points) Let $S = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 16 \end{bmatrix} \right\}$. Find a basis for S.

6. (5 points) Let $S = \{\vec{x} \in \mathbb{R}^3 | \langle 1, 0, 2 \rangle \cdot (\vec{x} \times \langle 2, 1, 3 \rangle) = 0\}$. Is S a subspace of $\mathbb{R}^3$? Justify your answer.

7. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_1 + x_2 - x_3 \\ x_1 - x_2 - x_3 \end{bmatrix}.$$

(a) (10 points) Find the matrix of linear transformation for T.

(b) (5 points) Find all vectors $\vec{x} \in \mathbb{R}^3$ such that $T(\vec{x}) = \vec{x}$.

8. Let

$$A = \begin{bmatrix} 1 & 3 & -3 \\ 3 & 1 & -3 \\ -3 & -3 & 1 \end{bmatrix}.$$

(a) (5 points) Find the eigenvalues of A. (Hint: $\lambda = 7$ is an eigenvalue with multiplicity 1.)

(b) (5 points) Find the eigenvectors of A.

(...continued)

(c) (5 points) Find matrices P and D such that $A = PDP^t$.

(d) (5 points) Use the information above to find A^3.

9. (5 points) For the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 & 6 \\ 0 & 3 & 5 & 7 & 4 & 6 \\ 0 & 6 & 9 & 12 & 7 & 10 \end{bmatrix}$$

with reduced row echelon form,

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find all vectors such that $A\vec{x} = \vec{0}$.

10. (5 points) Compute the determinant of

$$\begin{bmatrix} 4 & 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 & 4 \end{bmatrix}$$

11. (a) (5 points) Find an equation of the plane passing through these three points: $(1, 1, 0), (-1, 3, -2)$, and $(2, 1, 0)$.

    (b) (5 points) Find the symmetric equation of the line of intersection between the plane $2x-3y+2z = 1$ and the plane $2x + 3y + z = 6$. (Hint: for a point to be in the intersection, it must be on both planes.)