

## BISHOP'S UNIVERSITY

MATH 108: FINAL EXAM

Fall 2021

Name:	
Student #:	

- Prepare neat solutions. Briefly justify your work, that is, make your reasoning clear.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A Casio fx260-solar or Casio fx260-solar II calculator is permitted. No other aids are permitted.
- Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.

Page	Points	Score
2	20	
3	15	
4	15	
5	15	
6	15	
7	10	
8	10	
9	5	
10	15	
Total:	120	
5 6 7 8 9	15 15 10 10 5 15	

1. Consider the following matrices:

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 1 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (5 points)  $3A + 2C^tB^t$ 

(b) (5 points)  $A^2 - 5A + I$ 

(c) (5 points) 3DB + 10I

(d) (5 points)  $((BC)^t)^{-1}$ 

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 1 \\ 0 & 2 & -2 \end{bmatrix}.$$

(a) (3 points) Find the determinant of A.

(b) (10 points) Find the inverse of A.

(c) (2 points) Solve the following system of equations:  $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ 

3. (10 points) For what value of k, if any, will the system have (a) no solution, (b) a unique solution, (c) infinitely many solutions.

$$28x + 5ky = 2$$

$$7kx + 45y = -3$$

4. (5 points) Let A be a  $5 \times 5$  matrix satisfying  $A^2 = -A^{-1}$ . Find all possible values of  $\det(A)$ .

5. (10 points) Let  $S = \text{span } \left\{ \begin{bmatrix} 1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\2\\4 \end{bmatrix}, \begin{bmatrix} 1\\3\\9 \end{bmatrix}, \begin{bmatrix} 1\\4\\16 \end{bmatrix} \right\}$ . Find a basis for S.

6. (5 points) Let  $S = \{\vec{x} \in \mathbb{R}^3 | \langle 1, 0, 2 \rangle \cdot (\vec{x} \times \langle 2, 1, 3 \rangle) = 0 \}$ . Is S a subspace of  $\mathbb{R}^3$ ? Justify your answer.

7. Let  $T: \mathbb{R}^3 \to \mathbb{R}^3$  be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_1 + x_2 - x_3 \\ x_1 - x_2 - x_3 \end{bmatrix}.$$

(a) (10 points) Find the matrix of linear transformation for T.

(b) (5 points) Find all vectors  $\vec{x} \in \mathbb{R}^3$  such that  $T(\vec{x}) = \vec{x}$ .

8. Let

$$A = \begin{bmatrix} 1 & 3 & -3 \\ 3 & 1 & -3 \\ -3 & -3 & 1 \end{bmatrix}.$$

(a) (5 points) Find the eigenvalues of A. (Hint:  $\lambda=7$  is an eigenvalue with multiplicity 1.)

(b) (5 points) Find the eigenvectors of A.

 $(\dots continued)$ 

(c) (5 points) Find matrices P and D such that  $A = PDP^t$ .

(d) (5 points) Use the information above to find  $A^3$ .

9. (5 points) For the matrix

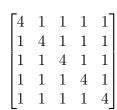
$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 & 6 \\ 0 & 3 & 5 & 7 & 4 & 6 \\ 0 & 6 & 9 & 12 & 7 & 10 \end{bmatrix}$$

with reduced row echelon form,

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find all vectors such that  $A\vec{x} = \vec{0}$ .

10. (5 points) Compute the determinant of



11. (a) (5 points) Find an equation of the plane passing through these three points: (1,1,0), (-1,3,-2), and (2,1,0).

(b) (5 points) Find the symmetric equation of the line of intersection between the plane 2x-3y+2z=1 and the plane 2x+3y+z=6. (Hint: for a point to be in the intersection, it must be on both planes.)