Bishop's University

Math 108: Final Exam
Fall 2023

Name: ____________________

Student #: ____________________

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	15	
4	15	
5	15	
6	10	
7	10	
8	15	
Total:	100	

1. Consider the following matrices:

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 1 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (5 points) BDC

(b) (5 points) $A^2 - 6A + I$

(c) (5 points) $3DB + 10I$

(d) (5 points) $(A^{-1}B)^T$

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

(a) (2 points) Find the trace of A.

(b) (5 points) Find the eigenvalues of A.

(c) (8 points) Find the unit eigenvectors of A.

3. (10 points) For what value(s) of k, if any, will the system have (a) no solution, (b) a unique solution, (c) infinitely many solutions.

$$\begin{aligned} -2x + y - kz &= 6 \\ x + 2y - z &= 3 \\ kx + y + 2z &= -12 \end{aligned}$$

4. (5 points) Let $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, ad - bc \neq 0 \right\}$. Is S non-empty? Is S closed under scalar multiplication? Is S closed under addition? Justify your answers.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_2 + x_3 \\ x_1 + 2x_2 - x_3 \\ x_1 - x_2 + 2x_3 \end{bmatrix}.$$

(a) (10 points) Find the matrix of linear transformation for T.

(b) (5 points) Find all vectors $\vec{x} \in \mathbb{R}^3$ such that $T(\vec{x}) = \vec{x}$.

6. (10 points) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Use diagonalization to compute A^7.

7. (10 points) For the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 & 6 \\ 2 & 0 & 4 & 5 & 6 & 7 \\ 3 & 0 & 5 & 6 & 7 & 8 \end{bmatrix}$$

with reduced row echelon form,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let S be the set of all vectors $\vec{x}$ such that $A\vec{x} = \vec{0}$. Find a basis for S.

8. (5 points) Compute the determinant of

$$\begin{bmatrix} 4 & 1 & 1 & 1 & -1 \\ 1 & 4 & 1 & 1 & -1 \\ 1 & 1 & 4 & 1 & -1 \\ 1 & 1 & 1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

9. (a) (5 points) Find an equation of the plane passing through these three points: $(1, 1, 0), (-1, 3, -2)$, and $(2, 1, 0)$.

   (b) (5 points) Find the distance from the point $(2, 5, -2)$ to the plane in part(a).