Bishop's University

Math 190: Exam
Fall 2015

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- Only a Casio fx260-solar calculator is permitted.

- This test is 180 minutes in length.

- Do not remove any pages from this test.

- The back of each page may be used for scrap paper.

Page	Points	Score
2	25	
3	20	
4	35	
5	25	
6	10	
7	20	
8	20	
9	25	
Total:	180	

1. (5 points) Rationalize the denominator of $\dfrac{7}{5 + \sqrt{11}}$.

2. (10 points) Briefly explain why the statement

$$\left(x^3 y - x^2 y^2\right)^3 = x^9 y^3 - x^6 y^6$$

is not true for all values of x and y. Write the correct simplified expansion of $\left(x^3 y - x^2 y^2\right)^3$, and evaluate when $x = 3$ and $y = 2$.

3. (10 points) Solve $\dfrac{6}{5q + 2} = \dfrac{2q}{1 - q} - 1$.

4. (10 points) Fully simplify $\dfrac{(a-b)^4 - (a+b)^4}{(a-b)^3 - (a+b)^3}$, remembering to write the answer in reduced form.

5. (10 points) Fully simplify the following expression, remembering to write the answer in reduced form with both the numerator and denominator expanded.

$$\frac{\left(\dfrac{1}{x+1} - \dfrac{1}{x^2-1}\right)}{\left(\dfrac{1}{x^2-1} + \dfrac{1}{x^3-1}\right)}$$

6. (15 points) The equation $6t^3 + 29t^2 + 46t + 24 = 0$ has three solutions. Use the fact that one of the solutions is $t = -2$ to factor $6t^3 + 29t^2 + 46t + 24$.

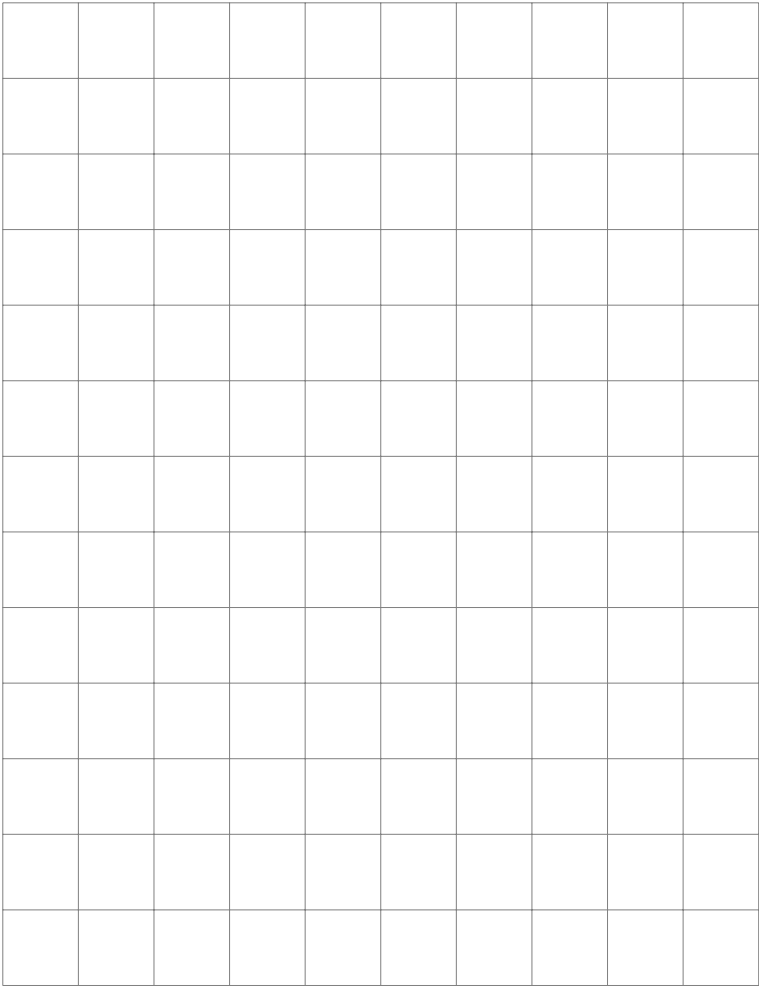7. Solve the following inequalities, and write the solution in interval notation.

   (a) (10 points) $\dfrac{2}{3t + 1} > 6$
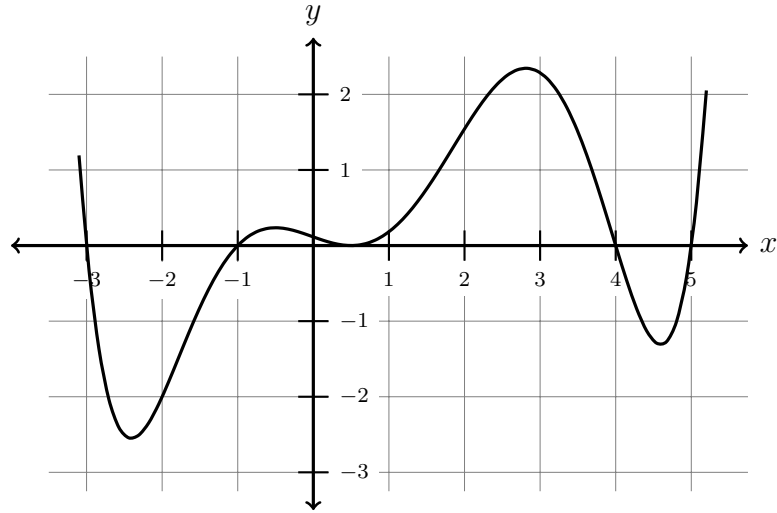
   (b) (10 points) $s^4 - 10s^2 + 9 \leq 0$

8. (10 points) Write the domain of $f(x) = \dfrac{\sqrt{2 + x - x^2}}{x^2 - 1}$ using set-builder notation.

9. (15 points) Fill in the table of values for $f(x) = -\dfrac{1}{2}x^3 - x^2 + \dfrac{5}{2}x + 3$ and plot the graph of f. Remember to properly place and scale the axes.

| x	-4	-3	-2	-1	0	1	2	3
$f(x)$								

10. (10 points) Consider the graph of the polynomial $p(x)$.

Fill in the blank or circle the answer which makes the sentence true.

(a) The degree of p is   (even / odd)

(b) The leading coefficient of p is   (positive / negative)

(c) The degree of p must be at least _____.

(d) The value of $p(-2)$ is _____.

(e) The number of solutions of $p(x) = 2$ is _____.

11. Consider the functions, f and g whose graphs are given below.

$$y = f(x) \qquad\qquad\qquad y = g(x)$$
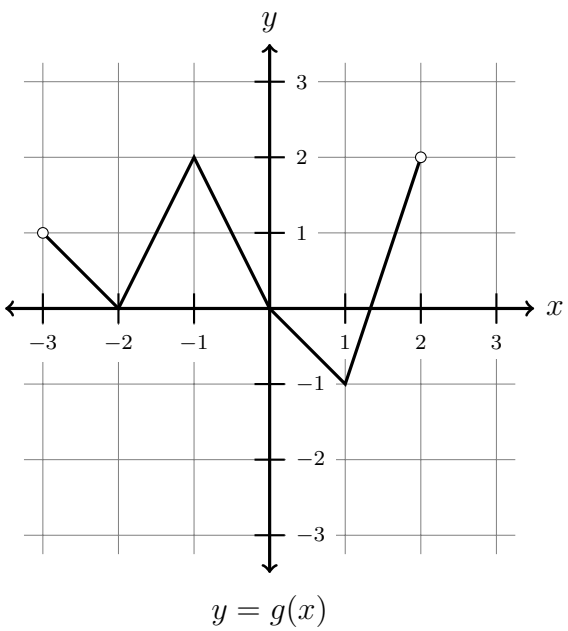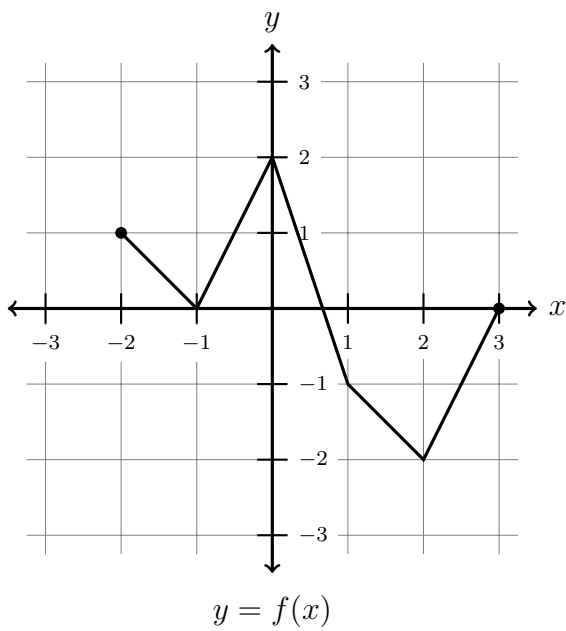
(a) (10 points) Sketch the graph for $y = (f + 2g)(x)$. Remember to place and scale the axes appropriately.

(b) (3 points) Evaluate $(f \circ g)(-1)$

(c) (7 points) Find the domain of $\dfrac{g}{f}$.

12. (14 points) Find the inverse of $f(x) = \dfrac{7x + 2}{2 - 3x}$ and find the domain of f^{-1}.

13. (6 points) Using the diagram below, identify the following functions:
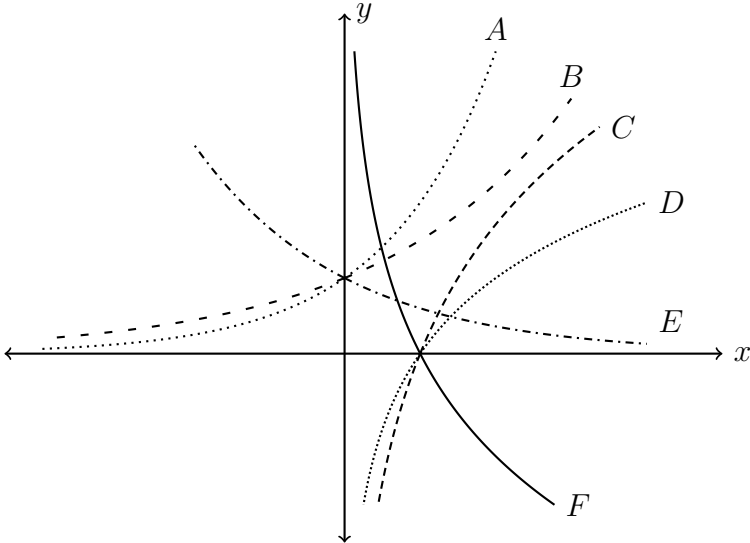
(a) $y = \left(\frac{3}{2}\right)^x$: _____   (d) $y = \log_{\frac{1}{2}} x$: _____

(b) $y = e^x$: _____   (e) $y = \log x$: _____

(c) $y = \left(\frac{3}{5}\right)^x$: _____   (f) $y = \log_2 x$: _____

14. (a) (5 points) Write $\log_2(x + 2) + \log_2(x - 3) - \log_2(3x)$ as a single logarithm.

(b) (5 points) Write $\log_2(x + 2) + \log_2(x - 3) - \log_2(3x) = 3$ in exponential form. (Hint: use result in part (a)).

(c) If $\log_a 2 = 0.301$, $\log_a 3 = 0.477$, and $\log_a 5 = 0.699$ evaluate:

   i. (5 points) $\log_a 30$

   ii. (5 points) $\log_a \dfrac{10}{9}$

   iii. (5 points) $\log_a 0.005$