Bishop's University

Math 191: Final Exam
Fall 2015

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- This test is 180 minutes in length.

- Do not remove any pages from this test.

- The back of each page may be used for scrap paper.

Page	Points	Score
2	25	
3	20	
4	25	
5	25	
6	30	
7	15	
8	15	
9	15	
Total:	170	

1. (15 points) Evaluate the following limits or state why the limit does not exist. If the limit is infinite, state whether is it positive or negative infinity. L'Hôpital's Rule is not permitted.

(a) $\displaystyle\lim_{v \to -2} \frac{v^2 - 4}{v^3 + 8}$

(b) $\displaystyle\lim_{x \to -3^-} \frac{x^3 + 9}{x + 3}$

(c) $\displaystyle\lim_{\theta \to 0} \frac{9\cot(3\theta)}{8\cot(4\theta)}$

2. (10 points) For what values of a and b is the function

$$f(x) = \begin{cases} x^2 + 2x - 2 & x < -2 \\ ax + b & -2 \le x < 3 \\ 9x - x^2 & x \ge 3 \end{cases}$$

continuous?

3. (a) (5 points) Show that if $-1 < x < 5$ then $\left|x^2 - 4\right| < 7\left|x - 2\right|$

(b) (5 points) Using the formal definition of a limit, prove that $\lim_{x \to 2} x^2 = 4$.

4. (10 points) Use the limit definition of derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x + 1}$.

5. Find the indicated derivative. DO NOT SIMPLIFY!

(a) (5 points) $z = \dfrac{1 - 2t + 3t^2}{1 + t}$, $\qquad\qquad z'$

(b) (5 points) $y = \dfrac{\sin x}{\sin x + \cos x}$, $\qquad\qquad \dfrac{dy}{dx}$

(c) (5 points) $f(x) = \sinh x \tanh x$, $\qquad\qquad f'(x)$

(d) (5 points) $g(x) = \cos^{-1}(\sqrt{x^2 - 1})$, $\qquad\qquad g'(x)$

(e) (5 points) $w = \ln(x^3 e^{5x-1})$, $\qquad\qquad w'$

6. (5 points) Evaluate the limit: $\displaystyle\lim_{x \to 1^+} \frac{4 - 4x}{\sqrt{x^2 - 1}}$.

7. (10 points) Verify that $y = e^{-x} \sin 2x$ is a solution of the differential equation $y'' + 2y' + 5y = 0$

8. (10 points) Show that $f(x) = x^3 + 3x^2 + 6x + 9$ has exactly one real root.

9. (10 points) Use logarithmic differentiation to find $f'(x)$ where

$$f(x) = \left(\sqrt{\frac{x^2 e^x (5x^3 + 3x + 7)^4}{\sin^2 x \cos^2 x}} \right)^3$$

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10. (10 points) The point $(3, 3)$ lies on the curve $x^3 + y^3 = 6xy$. Is the curve concave up or concave down near this point?

11. (10 points) Use the method of bisection to find a solution of $\cos x = x$ in an interval of length $\frac{1}{16}$

12. Let $f(x) = \dfrac{x}{x^2 + 1}$. Given that $f'(x) = \dfrac{1 - x^2}{(x^2 + 1)^2}$ and $f''(x) = \dfrac{2x(x^2 - 3)}{(x^2 + 1)^3}$, answer the following questions:

(a) (2 points) What is the domain of f?

(b) (3 points) What are the x-intercepts and y-intercepts of the graph $y = f(x)$?

(c) (3 points) Is f even, odd, periodic, or not symmetric?

(d) (3 points) What the horizontal and vertical asymptotes of f?

(e) (4 points) What are the intervals of increase and decrease for f?

(f) (4 points) What are the intervals of concavity for f?

(g) (4 points) Classify the critical points of f.

(h) (7 points) Sketch the graph of $y = f(x)$ using the information above, remembering to properly place and scale the axes.
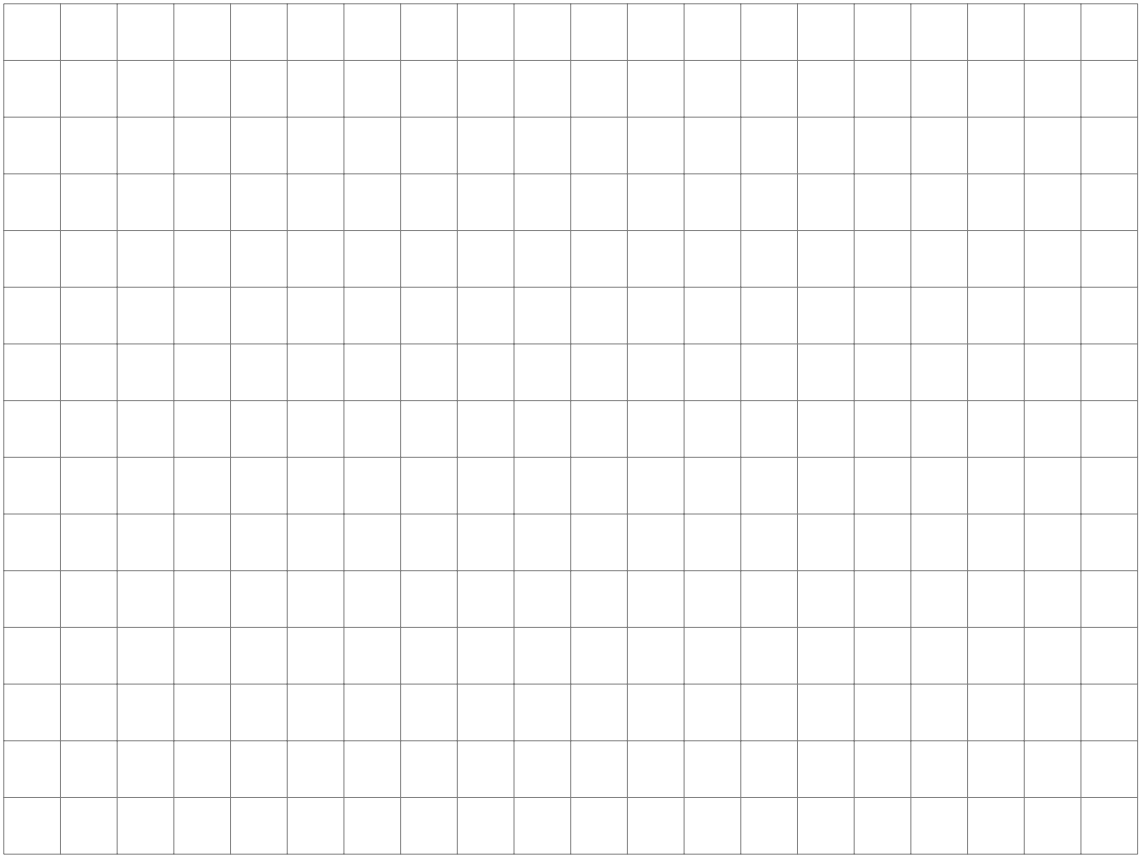
13. A wire 20 cm long is cut into two pieces. One piece is bent into the shape of a square and the other piece is bent into the shape of a circle.

   (a) (10 points) How should the wire be cut to minimize the total area enclosed by the shapes?

   (b) (5 points) Should the wire be cut if the total enclosed area is to be a maximum?