Bishop's University

Math 191/198: Final Exam
Fall 2020

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- The back of each page may be used for scrap paper.

- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.

- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	28	
3	12	
4	10	
5	10	
6	10	
7	11	
8	12	
9	7	
10	10	
Total:	110	

1. (28 points) Let u be a differentiable function of x. For the following functions, find $\dfrac{dy}{dx}$.

$y = a^u$	$y = \log_a u$
$y = \sin u$	$y = \csc u$
$y = \cosh u$	$y = \text{sech } u$
$y = \tan^{-1} u$	$y = \sec^{-1} u$
$y = \cot u$	$y = \tan u$
$y = \tanh^{-1} u$	$y = \coth^{-1} u$
$y = u^r$	$y = e^u$

2. (12 points) Evaluate the following limits or state why the limit does not exist. If the limit is infinite, state whether is it positive or negative infinity. L'Hôpital's Rule is not permitted.

   (a) $\displaystyle\lim_{v\to 2}\frac{v^2-4}{v^3-8}$

   (b) $\displaystyle\lim_{x\to -3^+}\frac{x^3+9}{x+3}$

   (c) $\displaystyle\lim_{\theta\to 0}\left(\frac{\cot(3\theta)}{\cot(4\theta)}\right)^3$

   (d) $\displaystyle\lim_{x\to 1}\frac{x^2+2x+3}{x+2}$

3. Let $f(x) = \dfrac{1}{2x + 1}$.

  (a) (8 points) Use the limit definition of derivative to find $f'(x)$.

  (b) (2 points) Find an equation of the tangent line to $y = f(x)$ at $x = -1$.

4. (10 points) For the following functions, find $\dfrac{dy}{dx}$. DO NOT SIMPLIFY!

(a) $y = \dfrac{x^2 e^{\cos x}}{1 - x^2}$

(b) $y = \sqrt{x^x (3x^4 + 5x + 1)^3 \sin^7(13x)}$

5. (10 points) The point $(2, -2)$ lies on the curve $x^4 + x^2 y^2 + y^3 = 24$. Is the curve concave up or concave down near this point?

6. Let $f(x) = \dfrac{-x^2 + 4x - 4}{x^2 + 4}$. Given that $f'(x) = \dfrac{16 - 4x^2}{(x^2 + 4)^2}$ and $f''(x) = \dfrac{8x(x^2 - 12)}{(x^2 + 4)^3}$, answer the following questions:

(a) (2 points) What is the domain of f?

(b) (3 points) What are the x-intercepts and y-intercepts of the graph $y = f(x)$?

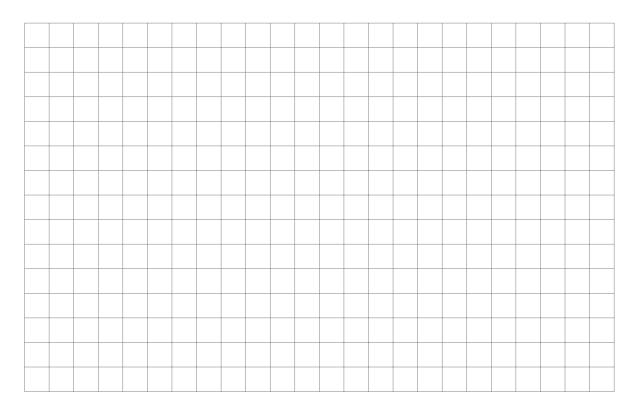(c) (3 points) Is f even, odd, periodic, or not symmetric?

(d) (3 points) What are the horizontal and vertical asymptotes of f?

(e) (4 points) What are the intervals of increase and decrease for f?

(f) (4 points) What are the intervals of concavity for f?

(g) (4 points) Classify the critical points of f.

(h) (7 points) Sketch the graph of $y = f(x)$ using the information above, remembering to properly place and scale the axes.

7. (10 points) Consider a right angle triangle with sides of length $3, 4$, and 5. Find the area of the largest (in terms of area) rectangle that can fit inside the triangle with one of its corners being the right angle of the triangle. (see diagram)