Bishop's University

Math 191: Final Exam
Fall 2022

Name: _____

Student #: _____

- This test is 180 minutes in length.

- All answers must be written in the space provided.

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- You are permitted to use one (1) Authorized Memory Book and a Casio fx-260 Solar (II) calculator.

Page	Points	Score
2	20	
3	15	
4	10	
5	5	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. For the following functions, find $\dfrac{dy}{dx}$. **DO NOT SIMPLIFY!**

(a) (5 points) $y = \cos(x)\cosh(2x)$

(b) (5 points) $y = (\sin(x) + 1)^x$

(c) (5 points) $y = \tan^{-1}\left(\dfrac{1}{x}\right)$

(d) (5 points) $y = \log_{10}(5x^2 + 3x - 2\pi^2)$

2. Evaluate the following limits or state why the limit does not exist. If the limit is infinite, state whether is it positive or negative infinity. L'Hôpital's Rule is not permitted.

(a) (5 points) $\displaystyle\lim_{x\to-3}\frac{x^2 + x - 6}{x^2 + 4x + 3}$

(b) (5 points) $\displaystyle\lim_{x\to0^+}\frac{x + \sqrt{x}}{\sqrt{x}}$

(c) (5 points) $\displaystyle\lim_{x\to0}\frac{4x\cot^2(7x)}{\csc(3x)}$

3. (10 points) Let $f(x) = 2x^2 - 3x + 5$. Use the **LIMIT DEFINITION** of the derivative at a point to evaluate $f'(-3)$.

4. (5 points) If $f(1) = 10$ and $f'(x) \geq 2$ for $x \geq 0$, what is the smallest possible value of $f(4)$? Justify your answer using the Mean Value Theorem.

5. (10 points) Consider the function $g(x) = x(x^2+1)(x-3)$. Find the x-coordinate of the point(s) where the tangent line is parallel to the line $y = -3x + 1$.

6. (10 points) A Ferris wheel with a radius of 10 meters is rotating at a rate of one revolution every two minutes. How fast is a rider rising when his seat is 16 meters above the ground?

7. (10 points) A hockey team plays in an arena with a seating capacity of 15,000 spectators. With ticket prices at \$12, average attendance at a game has been 11,000. A market survey indicates that for each dollar that ticket prices are lowered, the average attendance will increase by 1000. How should the owners of the team set ticket prices to maximize their revenue from ticket sales?

8. (10 points) Using the grid provided, sketch the graph of the function $f(x) = e^{-x^2}$. State its domain, intercepts, asymptotes, intervals of increase and decrease, critical point(s), intervals of concavity, and inflection point(s). You may use the fact that $f'(x) = -2xe^{-x^2}$ and $f''(x) = (4x^2 - 2)e^{-x^2}$
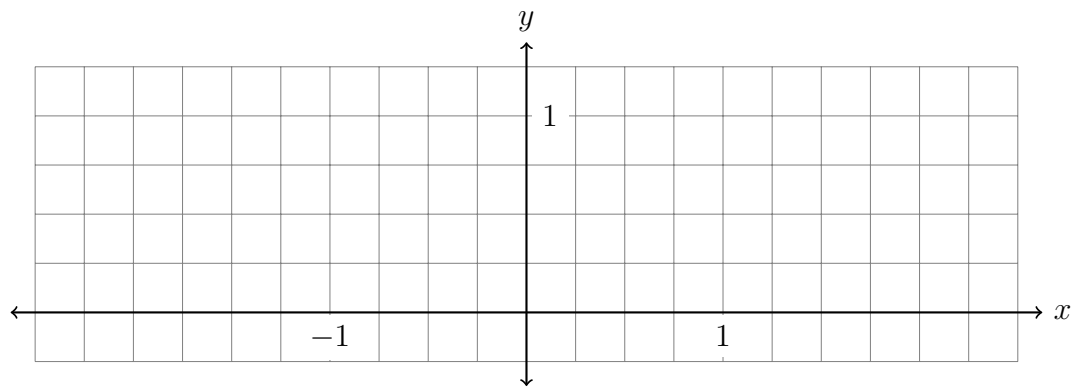
9. (10 points) Consider the following implicitly defined curve:

$$(x^2 + y^2 - 4)^3 = 108y^2 - 243$$

Find an equation of the tangent line at the point $(-2, -3)$.