Bishop's University

Math 192/199: Final Exam
Winter 2021

Name: ______________________________

Student #: ______________________________

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	12	
3	12	
4	12	
5	8	
6	18	
7	10	
8	10	
9	14	
10	13	
11	6	
Total:	115	

1. Evaluate the following integrals:

   (a) (4 points) $\int \left(x - \frac{1}{x}\right)^2 dx$

   (b) (4 points) $\int (x^e + e^3 + 3^x) dx$

   (c) (4 points) $\int_0^2 x^2\sqrt{x^3+1}dx$

(d) (4 points) $\int x \sinh(x^2) dx$

(e) (4 points) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^4(2x) \cos(2x) dx$

(f) (4 points) $\int_{-\pi/4}^{\pi/4} \frac{\sin 2x}{\cos x} dx$

(g) (4 points) $\int x^3 \ln(x) dx$

(h) (4 points) $\int_0^1 \frac{2x+3}{x+1} dx$

(i) (4 points) $\int \frac{1}{x^2 - 4x + 13} dx$

(j) (4 points) $\displaystyle\int \frac{x^2}{(9-x^2)^{3/2}}dx$

(k) (4 points) $\displaystyle\int_0^{\frac{\pi}{4}} \sec^3(x)\tan^5(x)dx$

2. Evaluate the following integrals:

   (a) (6 points) $\displaystyle\int \frac{3}{\sqrt{8-2x-x^2}}dx$

   (b) (6 points) $\displaystyle\int \frac{8-3x}{x(x-2)^2}dx$

   (c) (6 points) $\displaystyle\int \frac{5x^2-4x-3}{(2x-1)(x^2+1)}dx$

3. (5 points) Use the limit **definition of the definite integral (Riemann sum)** to evaluate $\int_0^2 (x^2 + 3x)dx$

4. (5 points) Find the arclength of the curve $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{3}$.

5. (a) (5 points) Give one or more definite integrals that describe the area of the region bounded by the curves $y^2 = 2 - x$ and $y = x$. Sketch the region. **DO NOT EVALUATE THE INTEGRAL**.

(b) (5 points) Give one or more definite integrals that describe the volume of the solid obtained by rotating the region bounded by $y = \cos x$, $y = 0$, $0 \leq x \leq \frac{\pi}{2}$ about the line $y = -1$. Sketch the region. **DO NOT EVALUATE THE INTEGRAL**.

6. (4 points) Find the average value of the function $f(x) = |x^3|$ on the interval $[-2, 1]$.

7. Determine whether the following improper integrals are convergent or divergent.
If convergent, find the value.

(a) (5 points) $\displaystyle\int_0^\infty \frac{x}{e^{x^2}} \; dx$

(b) (5 points) $\displaystyle\int_0^2 \frac{1}{\sqrt{4-x^2}} \; dx$

8. (4 points) Find the error bound for Simpson's Rule with $n = 10$ for $\displaystyle\int_0^4 e^x dx$.

9. True/False. Determine whether each of the statements are True or False.
Assume throughout that $f(x)$ and $g(x)$ are continuous functions with domain all real numbers, and that $a < b < c$ are real numbers. Indicate your answer by writing either **TRUE** or **FALSE** in the blanks provided. **The full word must be used**, not just T or F. **No marks will be given for T or F.**

(a) (1 point) If $f(x) \geq 0$ and is non-constant on $[a, b]$, then $\int_a^b f(x)dx > 0$: ....__________

(b) (1 point) If the graphs of f and g intersect midway between a and b, then
$\displaystyle\int_a^b (f(x) - g(x))dx = 0$: ....__________

(c) (1 point) $\displaystyle\int_a^b f(x)dx - \int_a^c f(x)dx = \int_b^c -f(x)dx$: ....__________

(d) (1 point) If $f(x) = \displaystyle\int_1^x \sin^2(t)dt$, then $f''(x) = \sin(2x)$: ....__________

(e) (1 point) $\displaystyle\int_{-1}^1 \frac{1}{x}\, dx = 0$: ....__________

(f) (1 point) $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\sin x}{(x^4+1)^2}\, dx = 0$: ....__________

(g) (1 point) $\displaystyle\int x\, f(x)\, dx = x \int f(x)\, dx$: ....__________

(h) (1 point) $\displaystyle\int_a^b \cos(x)\, dx = \int_a^{b+2\pi} \cos(x)\, dx$: ....__________

(i) (1 point) $\displaystyle\int_a^b f(x)dx$ is the area enclosed by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$:
....__________

10. (3 points) If $f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}} dt$, where $g(x) = \int_0^{\cos(x)} \left[1 + \sin(t^2)\right] dt$, find $f'(\frac{\pi}{2})$.

11. (3 points) Find the center of mass of a semicircular plate (half circle) of radius r with uniform density.