Bishop's University

Math 192: Deferred Exam
Winter 2022

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	16	
3	20	
4	14	
5	20	
6	6	
7	6	
8	6	
9	6	
10	6	
Total:	100	

1. (a) (4 points) Give the appropriate trigonometric substitution for $\int (x^2+9)^{\frac{3}{2}} dx$.

(b) (4 points) Write the form of the partial fraction decomposition of $\dfrac{3x^2+7x-13}{(x-1)^2(x+2)(x^2+4)}$, and DO NOT evaluate the coefficients.

(c) (4 points) Simplify $\cos(\tan^{-1} x)$ so that no trigonometric or inverse trigonometric functions are used.

(d) (4 points) Write an integral which is approximated by the Riemann sum $\displaystyle\sum_{i=1}^{n} \sinh\left(1+\frac{3i}{n}\right)\frac{3}{n}$, and DO NOT evaluate the integral

2. Evaluate the following integrals:

(a) (5 points) $\int (5-3x)^{10} dx$

(b) (5 points) $\int x \ln x dx$

(c) (5 points) $\int_0^{\pi} \sin^3 \theta d\theta$

(d) (5 points) $\int \frac{1}{\sqrt{x^2-4}} dx$

3. Evaluate the following integrals:

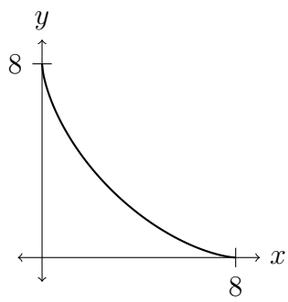
(a) (7 points) $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}dx$

(b) (7 points) $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}dx$

4. (10 points) Define $g(x) = \displaystyle\int_{x^2}^{x^3} \sin\left(\frac{\pi t^3}{2}\right) dt$. Find the equation of the tangent line to $y = g(x)$ at $x = -1$.

5. (10 points) Use the Trapezoid Rule with $n = 4$ subintervals to approximate the value of $\displaystyle\int_{-3}^{3} (x^3 - x)dx$.

6. **This question continues for several pages!**
Consider the astroid curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ with $0 \leq x \leq 8$ and $y \geq 0$. The graph is given in the diagram below. Let C denote the curve, and let R denote the region under the curve and above the x-axis for $0 \leq x \leq 8$. **It is highly recommended to complete part (i) of all the question before starting part(ii).**



(a) i. (4 points) In the box provided, write the integral representing the area of region R.

$A =$

ii. (2 points) Find the area of region R.

(b) i. (4 points) In the box provided, write the integral representing the volume of the solid generated by rotating the region R about the x-axis.

$V =$

ii. (2 points) Find the volume of the solid generated by rotating the region R about the x-axis.

(c) i. (4 points) In the box provided, write the integral representing the length of the curve C.

$L =$

ii. (2 points) Find the length of the curve C.

(d) i. (4 points) In the box provided, write the integral representing the lateral surface area of the solid generated by rotating the region R about the x-axis.

$SA =$

ii. (2 points) Find the lateral surface area of the solid generated by rotating the region R about the x-axis.

(e) i. (4 points) In the boxes provided, write the integral representing moments of the region R about the x-axis and the y-axis, assuming a uniform density of 1.

$M_y =$

$M_x =$

ii. (2 points) Find the center of mass of the region R (also called the centroid). Any use of symmetry must be justified.