



BISHOP’S UNIVERSITY

MATH 192: FINAL EXAM SPRING 2023

Name:

Student #:

- This test is 3 hours in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear*.
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop’s University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	20	
4	10	
5	10	
6	5	
7	5	
8	5	
9	5	
10	10	
Total:	90	

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1. (a) (5 points) Give the appropriate trigonometric substitution for $\int (x^2 + 9)^{\frac{3}{2}} dx$. **Do not evaluate the integral.**
- (b) (5 points) Write the form of the partial fraction decomposition of $\frac{3x^2 + 7x - 13}{(x - 1)^2(x + 2)(x^2 + 4)}$, and **DO NOT** evaluate the coefficients.
- (c) (5 points) Simplify $f(x) = \cos(\tan^{-1} x)$ so that no trigonometric or inverse trigonometric functions are used.
- (d) (5 points) Write an integral which is approximated by the Riemann sum $\sum_{i=1}^n \sinh\left(1 + \frac{3i}{n}\right) \frac{3}{n}$, and **do not evaluate the integral.**

2. Evaluate the following integrals:

(a) (5 points) $\int (5 - 3x)^{10} dx$

(b) (5 points) $\int x \ln x dx$

(c) (5 points) $\int_0^\pi \sin^3 \theta d\theta$

(d) (5 points) $\int \frac{1}{\sqrt{x^2 - 4}} dx$

3. Evaluate the following integrals:

(a) (5 points) $\int_0^1 \frac{x-4}{x^2-5x+6} dx$

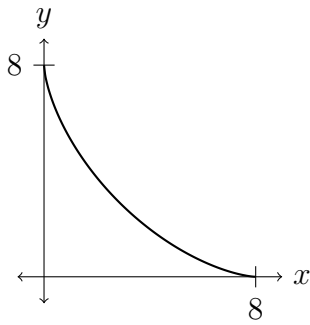
(b) (5 points) $\int_1^e \ln(x) dx$

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4. (5 points) Define $g(x) = \int_{x^2}^{x^3} \sin\left(\frac{\pi t^3}{2}\right) dt$. Find the equation of the tangent line to $y = g(x)$ at $x = -1$.

5. (5 points) The equation $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ defines an ellipse. We may assume that a and b are positive real numbers. Use the methods of this course to determine the area enclosed by the ellipse.

6. **This question continues for several pages! The figure will be included on each page for reference.**

Consider the astroid curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ with $0 \leq x \leq 8$ and $y \geq 0$. The graph is given in the diagram below. Let C denote the curve, and let R denote the region under the curve and above the x -axis for $0 \leq x \leq 8$. **It is highly recommended to complete part (i) of all the question before starting part(ii).**

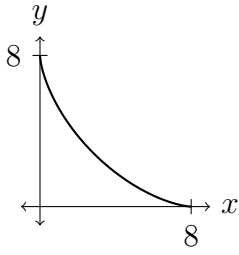


- (a) i. (4 points) In the box provided, write the integral representing the area of region R .

$A =$

- ii. (1 point) Find the area of region R .

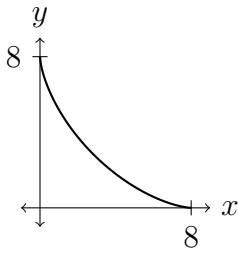
- (b) i. (4 points) In the box provided, write the integral representing the volume of the solid generated by rotating the region R about the x -axis.



$V =$

- ii. (1 point) Find the volume of the solid generated by rotating the region R about the x -axis.

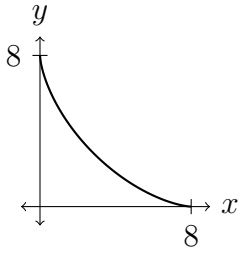
- (c) i. (4 points) In the box provided, write the integral representing the length of the curve C .



$L =$

- ii. (1 point) Find the length of the curve C .

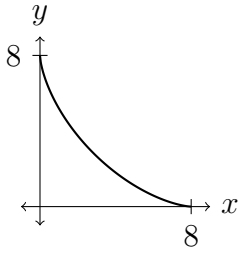
- (d) i. (4 points) In the box provided, write the integral representing the lateral surface area of the solid generated by rotating the region R about the x -axis.



$SA =$

- ii. (1 point) Find the lateral surface area of the solid generated by rotating the region R about the x -axis.

- (e) i. (8 points) In the boxes provided, write the integral representing moments of the region R about the x -axis and the y -axis, assuming a uniform density of 1.



$$M_y =$$

$$M_x =$$

- ii. (2 points) Find the center of mass of the region R (also called the centroid).