Bishop's University

Math 192: Final Exam
Winter 2023

Name: ____________________

Student #: ____________________

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	20	
4	10	
5	10	
6	5	
7	5	
8	5	
9	5	
10	10	
Total:	90	

1. (a) (5 points) Give the appropriate trigonometric substitution for $\int (x^2+9)^{\frac{3}{2}}\,dx$.

(b) (5 points) Write the form of the partial fraction decomposition of $\dfrac{3x^2+7x-13}{(x-1)^2(x+2)(x^2+4)}$, and DO NOT evaluate the coefficients.

(c) (5 points) Simplify $\cos(\tan^{-1} x)$ so that no trigonometric or inverse trigonometric functions are used.

(d) (5 points) Write an integral which is approximated by the Riemann sum $\displaystyle\sum_{i=1}^{n} \sinh\left(1+\frac{3i}{n}\right)\frac{3}{n}$, and DO NOT evaluate the integral

2. Evaluate the following integrals:

   (a) (5 points) $\int (5-3x)^{10} dx$

   (b) (5 points) $\int x \ln x dx$

   (c) (5 points) $\int_0^{\pi} \sin^3 \theta d\theta$

   (d) (5 points) $\int \frac{1}{\sqrt{x^2-4}} dx$

3. Evaluate the following integrals:

   (a) (5 points) $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}dx$

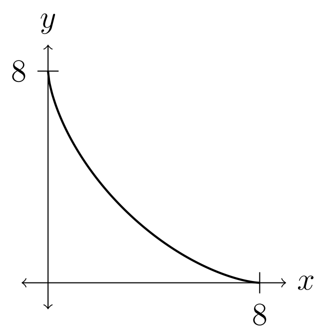
   (b) (5 points) $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}dx$

4. (5 points) Define $g(x) = \int_{x^2}^{x^3} \sin\left(\frac{\pi t^3}{2}\right) dt$. Find the equation of the tangent line to $y = g(x)$ at $x = -1$.

5. (5 points) The average value of a continuous function f on the interval $[a, b]$ is denote $\overline{f}$, and it satisfies the equation: $(b-a)\overline{f} = \int_a^b f(x)dx$.

   Let $T(t) = 20 + 12\sin\left(\frac{(t+6)\pi}{18}\right)\sin\left(\frac{(t-18)\pi}{24}\right)$ be the temperature in degrees Celsius t hours after midnight. This model is valid for $0 \leq t \leq 24$. Find the average temperature for that 24-hour period.

6. **This question continues for several pages! The figure will be included on each page for reference.**
Consider the astroid curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ with $0 \leq x \leq 8$ and $y \geq 0$. The graph is given in the diagram below. Let C denote the curve, and let R denote the region under the curve and above the x-axis for $0 \leq x \leq 8$. **It is highly recommended to complete part (i) of all the question before starting part(ii).**

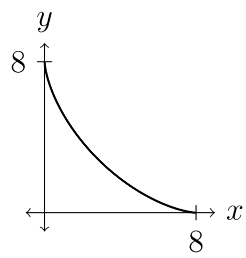


(a) i. (4 points) In the box provided, write the integral representing the area of region R.

$A =$

ii. (1 point) Find the area of region R.

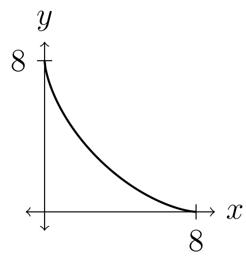
(b) i. (4 points) In the box provided, write the integral representing the volume of the solid generated by rotating the region R about the x-axis.



$V =$

ii. (1 point) Find the volume of the solid generated by rotating the region R about the x-axis.

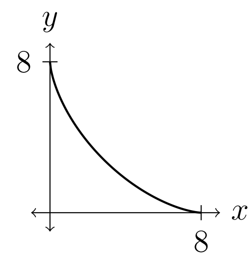
(c) i. (4 points) In the box provided, write the integral representing the length of the curve C.



$$L =$$

ii. (1 point) Find the length of the curve C.

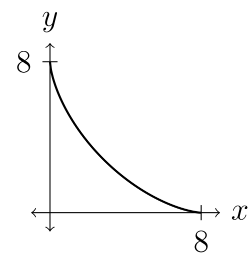
(d) i. (4 points) In the box provided, write the integral representing the lateral surface area of the solid generated by rotating the region R about the x-axis.



$$SA =$$

ii. (1 point) Find the lateral surface area of the solid generated by rotating the region R about the x-axis.

(e) i. (8 points) In the boxes provided, write the integral representing moments of the region R about the x-axis and the y-axis, assuming a uniform density of 1.



$M_y =$

$M_x =$

ii. (2 points) Find the center of mass of the region R (also called the centroid).