Bishop's University

Math 192: Final Exam
Winter 2023

Name: _____

Student #: _____

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	20	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Total:	100	

1. (a) (5 points) Give the appropriate trigonometric substitution for $\int (25x^2 - 49)^{\frac{5}{2}} dx$.

(b) (5 points) Write the form of the partial fraction decomposition of $\dfrac{3x^2 + 7x - 13}{(2x-1)(x-3)(x^2+3x+4)^2}$, and DO NOT evaluate the coefficients.

(c) (5 points) Simplify $\tan(\sin^{-1} x)$ so that no trigonometric or inverse trigonometric functions are used.

(d) (5 points) Write an integral which is approximated by the Riemann sum $\displaystyle\sum_{i=1}^{n} \ln\left(4 - \frac{2i}{n}\right)\frac{4}{n}$, and DO NOT evaluate the integral

2. Evaluate the following integrals:

(a) (5 points) $\int (3-5x)^7 dx$

(b) (5 points) $\int x\cos x dx$

(c) (5 points) $\int_0^{\frac{\pi}{6}} \sin(3\theta)\cos(4\theta)d\theta$

(d) (5 points) $\int \frac{x^2}{\sqrt{x^2-4}}dx$

3. Evaluate the following integrals:

(a) (5 points) $\displaystyle\int_0^1 \frac{x-4}{x^2+5x+6}dx$

(b) (5 points) $\displaystyle\int_2^\infty \frac{e^{-1/x^2}}{x^3}dx$

4. (5 points) Define $g(x) = \displaystyle\int_{x^2}^{x^3} \sin\left(\frac{\pi t^5}{2}\right) dt$. Find the equation of the tangent line to $y = g(x)$ at $x = -1$.

5. (5 points) Let $T(t) = 20 - 10\cos\left(\frac{\pi t}{12}\right)$ be the temperature (in °C) t hours after midnight. This model is valid for $0 \leq t \leq 24$. Find the average day-time temperature for the period of 6am to 9pm.

6. (10 points) Evaluate the integral $\int \frac{x^{1/2}}{x^{3/2}+1} dx$.

7. (10 points) Use the formula $\displaystyle\int \frac{dx}{x^2\sqrt{a^2-x^2}} = \frac{-\sqrt{a^2-x^2}}{a^2x} + C$, where a is a positive constant, to evaluate

$$\int \frac{\cos\theta}{\sin^2\theta\sqrt{9-4\sin^2\theta}}d\theta.$$

8. (10 points) Find the volume of the solid of revolution obtained by rotating the region bound by the curves $y = \tan(x^2)$, $x = \frac{\sqrt{\pi}}{2}$, and $y = 0$ about the y-axis.

9. (10 points) An equilateral plate is submerged vertically in water (density is $\rho = 1000$ kg/m^3) with one side at surface level. Each side of the plate is 4 meters in length. If gravity is given to be 10 m/s^2, calculate the hydrostatic force against on side of the plate.

Some useful integration formulas to carry around.

1. $\int u dv = uv - \int v du$
2. $\int u^n du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\displaystyle\int \frac{du}{u} = \ln|u| + C$
4. $\displaystyle\int a^u du = \frac{1}{\ln a} a^u + C$
5. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$
6. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$
7. $\displaystyle\int \sqrt{a^2-u^2} du = \frac{u}{a}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\left(\frac{u}{a}\right) + C$
8. $\displaystyle\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$
9. $\displaystyle\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$
10. $\displaystyle\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin(2u) + C$
11. $\displaystyle\int \sin^n u \, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u \, du$
12. $\displaystyle\int \cos^n u \, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u \, du$
13. $\int \tan u \, du = \ln|\sec u| + C$
14. $\int \cot u \, du = \ln|\sin u| + C$
15. $\int \tan^2 u \, du = \tan u - u + C$
16. $\int \cot^2 u \, du = -\cot u - u + C$
17. $\displaystyle\int \tan^n u \, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u \, du + C, \quad n \neq 1$
18. $\displaystyle\int \cot^n u \, du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u \, du + C, \quad n \neq 1$
19. $\int \sec u \, du = \ln|\sec u + \tan u| + C$
20. $\int \csc u \, du = \ln|\csc u - \cot u| + C$
21. $\displaystyle\int \sec^n u \, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u \, du + C, \quad n \neq 1$
22. $\displaystyle\int \csc^n u \, du = \frac{-1}{n-1}\csc^{n-2} u \cot u + \frac{n-2}{n-1}\int \csc^{n-2} u \, du + C, \quad n \neq 1$
23. $\int \sec u \tan u \, du = \sec u + C$
24. $\int \csc u \cot u \, du = -\csc u + C$
25. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$
26. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$
27. $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$