BISHOP'S UNIVERSITY

MATH 196: FINAL EXAM
FALL 2013

Name: _____

Student #: _____

Time: 3 hours

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- All dollar values must be given to the nearest penny, unless otherwise indicated.

- Do not remove any pages from this test.

- The back of each page may be used for scrap paper.

Useful Formulas

$$A = P \left(1 + \frac{r_n}{m}\right)^{mt}$$

$$r_e = \left(1 + \frac{r_n}{m}\right)^{m} - 1$$

$$S = R \left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt} - 1}{\frac{r_n}{m}} \right)$$

$$P = R \left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{-mt}}{\frac{r_n}{m}} \right)$$

$$S = R \left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt+1} - 1}{\frac{r_n}{m}} - 1 \right)$$

$$P = R \left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{1-mt}}{\frac{r_n}{m}} + 1 \right)$$

Page	Points	Score
2	18	
3	18	
4	12	
5	16	
6	15	
7	11	
8	10	
Total:	100	

1. Expand and simplify the following expressions, making sure to remove all brackets, combine like terms and avoid negative and fractional exponents:

   (a) (3 points) $(x+7)(x-7)$

   (b) (3 points) $(2\sqrt{n}+1)(\sqrt{n}-3)$

   (c) (3 points) $(2+\sqrt[3]{q})^3$

2. Write as a single fraction and simplify the following expressions, making sure to remove all brackets, combine like terms, cancel common factors, and avoid negative and fractional exponents:

   (a) (3 points) $\dfrac{3x+2}{x-1}+\dfrac{x-1}{x+1}$

   (b) (3 points) $\dfrac{1}{x}+\dfrac{1}{x+1}+\dfrac{1}{x+2}$

   (c) (3 points) $\dfrac{5}{\sqrt{x}+2}+\dfrac{3}{\sqrt{x}-2}$

3. Solve the following inequalities:

(a) (3 points) $3n + 2 < n - 1$

(b) (3 points) $5y - 7 \geq 2y + 5$

(c) (3 points) $|3x - 1| > \dfrac{6}{5}$

4. Evaluate the indicated function value.

(a) (2 points) $f(-2)$ where $f(x) = 3x^2 + 2x - 1$.

(b) (2 points) $h(3)$ where $h(r) = \dfrac{5r^2 - 3r + 1}{r - 4}$.

(c) (2 points) a_7 where $a_i = (-1)^i (3i - 1)^2$.

5. (3 points) Find the missing terms which fit the pattern.

$$2, 6, 11, 17, 24, 32, 41, 51, \underline{\quad}, \underline{\quad}, \underline{\quad}$$

6. (6 points) Find the missing terms which fit the pattern, write in summation notation, and evaluate the sum

$$100 + 91 + 82 + 73 + 64 + 55 + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

7. Evaluate, if possible,

(a) (2 points) $\sum_{k=1}^{50} (3k - 1)^2$

(b) (2 points) $\sum_{i=1}^{20} \frac{2^{i-1}}{75}$

(c) (2 points) $\sum_{n=1}^{\infty} 5 \left(\frac{1}{3}\right)^{n-1}$

8. Consider the line $5x - 7y = 105$.

  (a) (2 points) Find the x-intercept and the y-intercept.

  (b) (1 point) Find the slope.

  (c) (2 points) Write the slope-intercept equation for this line.

  (d) (2 points) Find the general form of the equation of the line through the point $(15, -10)$ which is perpendicular to the given line. The coefficient in front of the y must be 5.

9. (9 points) Samantha wants to start an annuity that will have a value of $10,000 at the end of five years. Bank A offers an ordinary annuity at 3% compounded monthly. Bank B offers an annuity due at 2.84% compounded quarterly. Which bank should Samantha choose to minimize her total payments? Remember to justify your answer.

10. Consider the following matrices:

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ -2 & 3 \\ 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

Evaluate, if possible,

(a) (3 points) $A + BC$

(b) (3 points) ABD

(c) (3 points) $DC + I$

(d) (3 points) $(BC)^{-1}$

11. (3 points) The matrix equation

$$\begin{bmatrix} 4 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

represents a system of equations. Write that system of equations. **Do not solve.**

12. (5 points) Solve the system of equations by the method of reduction.

$$\begin{cases} x + 2y + 10z = -14 \\ 2x + 3y - z = 5 \end{cases}$$

13. (6 points) A chemical manufacturer wishes to fill an order for 900 litres of a 25% acid solution. Solutions of 20% and 35% are in stock. How many litres of each solution must be mixed to fill the order?

14. Consider the system of equations

$$\begin{cases} x & + & y & + & z & = & 2 \\ x & & & - & z & = & -4 \\ x & - & y & - & 2z & = & 5 \end{cases}$$

(a) (1 point) Write the coefficient matrix for this system.

(b) (7 points) Find the inverse of the coefficient matrix.

(c) (2 points) Solve the system by using the inverse.