Bishop's University

Math 196: Final Exam
Fall 2014

Name: _____

Student #: _____

Time: 3 hours

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.

Useful Formulas

$$A = P\left(1 + \frac{r_n}{m}\right)^{mt}$$

$$r_e = \left(1 + \frac{r_n}{m}\right)^{m} - 1$$

$$S = R\left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt} - 1}{\frac{r_n}{m}}\right)$$

$$P = R\left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{-mt}}{\frac{r_n}{m}}\right)$$

$$S = R\left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt+1} - 1}{\frac{r_n}{m}} - 1\right)$$

$$P = R\left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{1-mt}}{\frac{r_n}{m}} + 1\right)$$

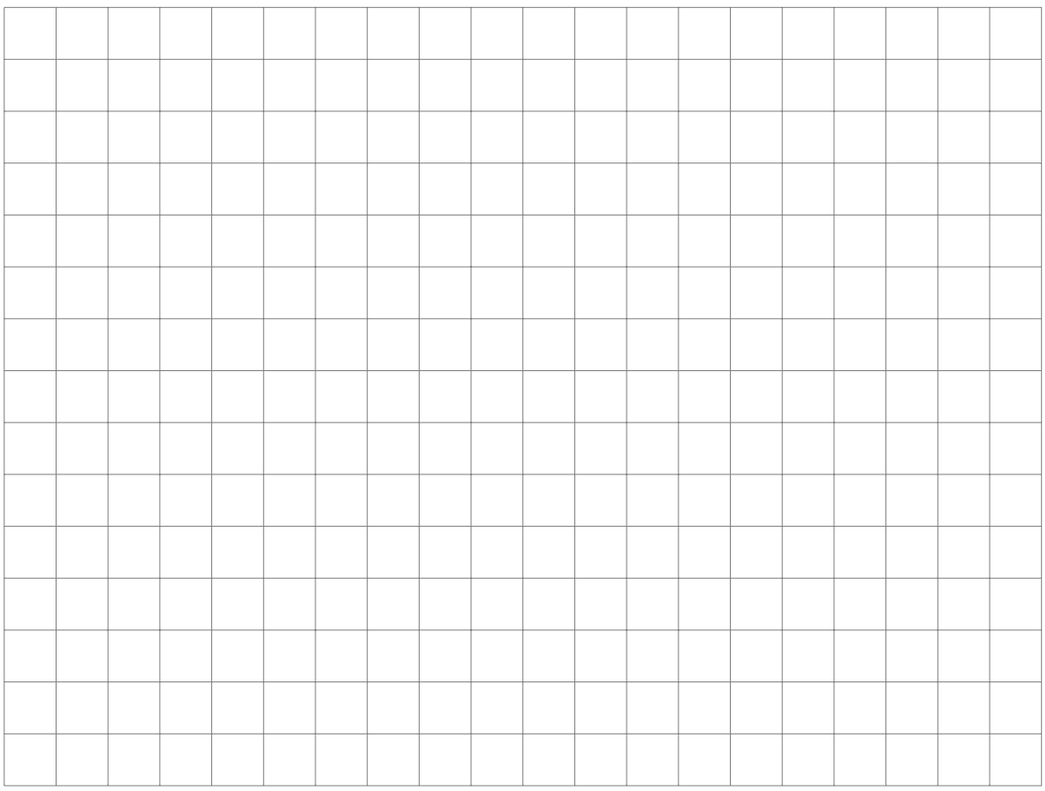
× \ +	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	0	2	3	4	5	6	7	8	9	10	11
2	0	2	4	5	6	7	8	9	10	11	12
3	0	3	6	6	7	8	9	10	11	12	13
4	0	4	8	12	8	9	10	11	12	13	14
5	0	5	10	15	20	10	11	12	13	14	15
6	0	6	12	18	24	30	12	13	14	15	16
7	0	7	14	21	28	35	42	14	15	16	17
8	0	8	16	24	32	40	48	56	16	17	18
9	0	9	18	27	36	45	54	63	72	18	19
10	0	10	20	30	40	50	60	70	80	90	20

Page	Points	Score
2	12	
3	15	
4	12	
5	15	
6	17	
7	9	
8	7	
9	12	
Total:	99	

1. (3 points) Solve the following inequality: $|2x + 3| > \frac{11}{4}$

2. (5 points) Sketch the region described by these inequalities, remembering to shade the **EXCLUDED** regions. Proper scaling and placement of axes will be taken into account in the grading.

$$\begin{cases} x - y \geq -5 \\ 2x + y \geq -2 \\ y \leq 5 \\ y \geq -4 \\ x \leq 4 \end{cases}$$



3. (4 points) Find the domain of $g(t) = \dfrac{\sqrt{4t + 20}}{t^2 - 9}$.

4. (6 points) For security reason, a company will enclose a rectangular area of 11,200 m^2 in the rear of its manufacturing plant. One side will be bounded by the building (and thus will not require a fence), and the other three sides will be fenced. If 300 feet of fencing is used, what will be the dimension of the rectangular area? **Note: There may be more than one correct answer.**

5. (3 points) Find the missing terms which fit the pattern.

$$10, 19, 37, 73, 145, \_\_\_\_, \_\_\_\_, \_\_\_\_$$

6. (6 points) Find the missing terms which fit the pattern, write in summation notation, and evaluate the sum

$$1 + 3 + 7 + 15 + 31 + 63 + 127 + 255 + \_\_\_\_ + \_\_\_\_ + \_\_\_\_$$

7. (3 points) Consider the recursive sequence given by $a_1 = 0$ and $a_{k+1} = 1 - 3a_k$ for $k \geq 1$. Find a_6.

8. (3 points) Evaluate the eighty-fourth (84^{th}) term of the sequence $(a_k)_{k=1}^{\infty} = \left(\frac{1}{k}\sum_{i=1}^{k} i\right)_{k=1}^{\infty}$.

9. Evaluate, if possible,

   (a) (3 points) $\displaystyle\sum_{k=1}^{12} k(13-k)$

   (b) (3 points) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{3}{4}\right)^{n-1}$

10. (3 points) A \$15,000 investment is made into an account that pays a nominal rate of 6% compounded monthly. After twelve years, is it reasonable to assume that there will not be more than \$25,000 in the account? Why or why not?

11. (6 points) The present value of a 25-year mortgage is \$200,000. If the nominal interest rate is 6% compounded monthly, with the payments made at the end of each month, write the simplified expression for the regular monthly payment. **You may use decimals in this answer, but you may not round your answer.**

12. (6 points) Write the 3×4 matrix, A, whose entries are given by $a_{ij} = ((-1)^{i+j})i + j$.

13. Consider the following matrices:

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ -2 & 3 \\ 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (3 points) $3A - 2BC$

(b) (3 points) DCA

(c) (3 points) $3DB + 10I$

(d) (3 points) $((BC)^T)^{-1}$

14. (5 points) A matrix A is called *magical* if $A^T = -A$. Prove that for any square matrix M, $M - M^T$ is magical.

15. (3 points) The matrix equation

$$\begin{bmatrix} 4 & -2 & 1 \\ 1 & -4 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

represents a system of equations. The augmented matrix in reduced form is

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{7} & 1 \\ 0 & 1 & \frac{1}{14} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Write the solution of the system of equations.

16. (6 points) A gardener has two fertilizers that contain different concentrations of nitrogen. One is 3% nitrogen by weight, and the other is 11% nitrogen by weight. How many pounds of each should she mix to obtain 20 pounds of a 9% concentration (by weight)?

17. (5 points) Solve the system of equations by the method of reduction.

$$\begin{cases} x + 3y - 2z = -2 \\ 2x + 7y + z = 3 \end{cases}$$

18. (2 points) A profit of 40% on the selling price of a product is equivalent to what percentage profit on the cost? **You may use decimals in your answer. If you do, you must round to one (1) decimal place.**

19. Consider the system of equations

$$\begin{cases} x & & - & z & = & 2 \\ & y & - & z & = & -4 \\ x & - & y & - & z & = & 5 \end{cases}$$

(a) (2 points) Write the coefficient matrix for this system.

(b) (7 points) Find the inverse of the coefficient matrix.

(c) (3 points) Solve the system by using the inverse.