Bishop's University

Math 196: Final Exam
Fall 2015

Name: _____

Student #: _____

Time: 3 hours

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Only a Casio fx260-solar calculator is permitted.
- All monetary values are to be given to the nearest penny unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.

Useful Formulas

$$A = P\left(1+\frac{r_n}{m}\right)^{mt}$$

$$r_e = \left(1+\frac{r_n}{m}\right)^{m} - 1$$

$$S = R\left(\frac{\left(1+\frac{r_n}{m}\right)^{mt}-1}{\frac{r_n}{m}}\right)$$

$$P = R\left(\frac{1-\left(1+\frac{r_n}{m}\right)^{-mt}}{\frac{r_n}{m}}\right)$$

$$S = R\left(\frac{\left(1+\frac{r_n}{m}\right)^{mt+1}-1}{\frac{r_n}{m}} - 1\right)$$

$$P = R\left(\frac{1-\left(1+\frac{r_n}{m}\right)^{1-mt}}{\frac{r_n}{m}} + 1\right)$$

Page	Points	Score
2	25	
3	30	
4	20	
5	20	
6	30	
7	20	
8	15	
9	20	
Total:	180	

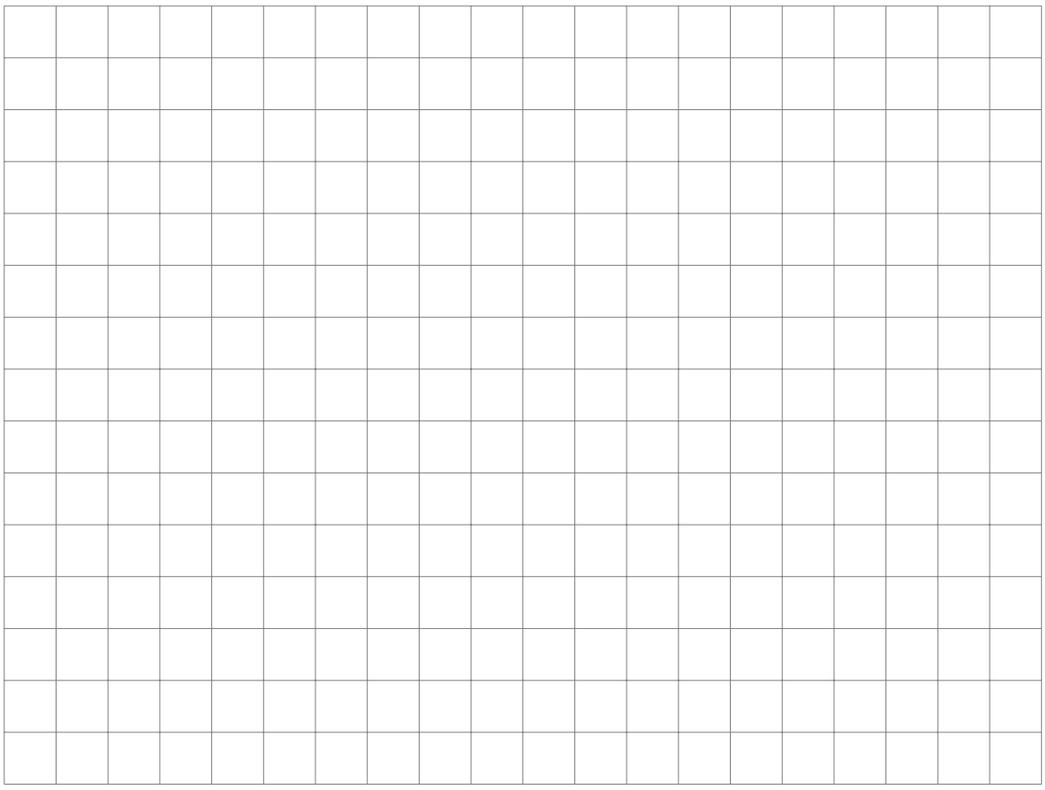
_____

Signature

1. (10 points) Solve the following inequality: $|3x + 4| > \frac{11}{7}$

2. (15 points) Sketch the region described by these inequalities, remembering to shade the **EXCLUDED** regions. Proper scaling and placement of axes will be taken into account in the grading.

$$\begin{cases} x + y & \leq & 6 \\ x + 3y & \geq & -9 \\ x - 3y & \leq & -3 \\ 5x - 9y & \geq & -45 \end{cases}$$



3. (5 points) The sales tax in Quebec is 14.975 %. If the cost of a purchase, including tax, is \$14,450.84, how much of it is tax? Remember to give the answer to the nearest penny.

4. (6 points) Find the missing terms which fit the pattern.

$$5, 9, 17, 33, 65, \_\_\_\_, \_\_\_\_, \_\_\_\_, 1025$$

5. (9 points) Find the missing terms which fit the pattern, write in summation notation, and evaluate the sum using the formulas taught in class.

$$3 + 6 + 12 + 24 + 48 + 96 + 192 + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + 3072$$

6. (10 points) Consider the recursive sequence given by $a_1 = 10$ and $a_k = 2a_{k-1} - k^2$ for $k \geq 2$. Find a_8.

7. (5 points) Evaluate the seventy-fifth (75^{th}) term of the sequence $(a_k)_{k=1}^{\infty} = \left(\dfrac{1}{k^2+k} \displaystyle\sum_{i=1}^{k} i^3 \right)_{k=1}^{\infty}$.

8. Evaluate, if possible,

   (a) (5 points) $\displaystyle\sum_{k=1}^{12} (k+1)(k-5)$

   (b) (5 points) $\displaystyle\sum_{n=1}^{\infty} 3\,(-3)^{1-n}$

9. (5 points) A \$15,000 investment is made into an account that pays a nominal rate of 6% compounded monthly. What is the minimum number of months the deposit must be left in the account to achieve a balance of at least \$29,000?

10. (10 points) A 25-year mortgage for $180,000 is granted to a new homeowner. If the nominal interest rate is 3% compounded monthly, with the payments made at the end of each month, what are the regular monthly payments? How much is paid in interest over the 25 years?

11. (10 points) Dr. Jones wishes to retire someday. After he retires, he estimates his life expectancy to be twenty years. Dr. Jones has calculated that he will need a monthly income of $2700 during his retirement. How much does he need to have in his RRSP (registered reitrement saving plan), which has an annual nominal rate of 1.5% compounded monthly, if he withdraws the whole monthly income at the beginning of each month? The RRSP is his only source of income.

12. (10 points) Write the 3×4 matrix, A, whose entries are given by $a_{ij} = ((-1)^{i+j})j + 2i$.

13. Consider the following matrices:

$$A = \begin{bmatrix} 5 & 1 \\ -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ -3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (5 points) $3A + 2BC$

(b) (5 points) DBA

(c) (5 points) $3CB + 10I$

(d) (5 points) $((BC)^T)^{-1}$

14. (5 points) A matrix M is called *equivocal* if $(M^{-1})^T = M$. For any pair of $n \times n$ equivocal matrices A and B, prove that AB is equivocal.

15. (10 points) A gardener has two fertilizers that contain different concentrations of nitrogen. One is 5% nitrogen by weight, and the other is 12% nitrogen by weight. How many pounds of each should she mix to obtain 20 pounds of a 7.8% concentration (by weight)?

16. (5 points) A profit of 75% on the selling price of a product is equivalent to what percentage profit on the cost? Give your answer as a percentage rounded to two decimal places.

17. (15 points) Solve the system of equations by the method of reduction.

$$\begin{cases} x_1 + 3x_2 - 2x_3 + x_4 = -2 \\ 2x_1 + 7x_2 + x_3 - 2x_4 = 3 \\ -x_1 - x_2 + 12x_3 + x_4 = -4 \end{cases}$$

18. Consider the system of equations

$$\begin{cases} x & & & + & z & = & 2 \\ & & y & + & z & = & -4 \\ x & + & y & - & z & = & -5 \end{cases}$$

(a) (5 points) Write the coefficient matrix for this system.

(b) (10 points) Find the inverse of the coefficient matrix using methods discussed in class.

(c) (5 points) Solve the system by using the inverse.