Bishop's University

Math 196: Final Exam
Winter 2015

Name:	_____
Student #:	_____
Time:	3 hours

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- Do not remove any pages from this test.

- The back of each page may be used for scrap paper.

Useful Formulas

$$A = P\left(1 + \frac{r_n}{m}\right)^{mt}$$

$$r_e = \left(1 + \frac{r_n}{m}\right)^m - 1$$

$$S = R\left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt} - 1}{\frac{r_n}{m}}\right)$$

$$P = R\left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{-mt}}{\frac{r_n}{m}}\right)$$

$$S = R\left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt+1} - 1}{\frac{r_n}{m}} - 1\right)$$

$$P = R\left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{1-mt}}{\frac{r_n}{m}} + 1\right)$$

+ ×	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	0	1	3	4	5	6	7	8	9	10	11
2	0	2	4	5	6	7	8	9	10	11	12
3	0	3	6	9	7	8	9	10	11	12	13
4	0	4	8	12	16	9	10	11	12	13	14
5	0	5	10	15	20	25	11	12	13	14	15
6	0	6	12	18	24	30	36	13	14	15	16
7	0	7	14	21	28	35	42	49	15	16	17
8	0	8	16	24	32	40	48	56	64	17	18
9	0	9	18	27	36	45	54	63	72	81	19
10	0	10	20	30	40	50	60	70	80	90	100

Page	Points	Score
2	12	
3	13	
4	12	
5	9	
6	17	
7	16	
8	11	
9	12	
Total:	102	

1. (5 points) Solve the following inequality: $\left| \dfrac{2x}{5} - \dfrac{4}{3} \right| \geq \dfrac{11}{7}$ and write the answer using set-builder notation.

2. (7 points) Sketch the region described by these inequalities, remembering to shade the **EXCLUDED** regions. Proper scaling and placement of axes will be taken into account in the grading.

$$\begin{cases} 2x - 3y & \leq & 6 \\ 4x + 3y & \leq & 12 \\ -2x + y & \geq & -8 \\ y & \geq & -8 \\ x & \geq & -14 \end{cases}$$
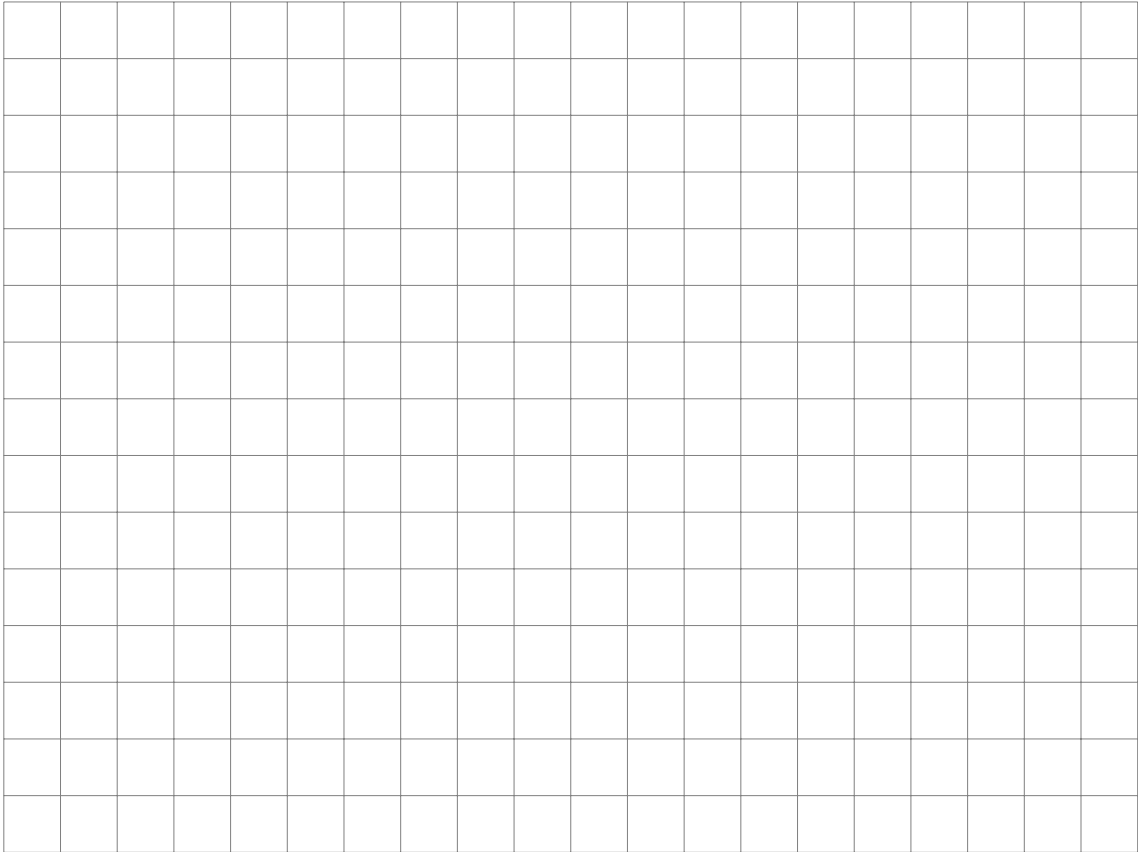
3. (6 points) A land investment company purchased a parcel of land for $7200. After having sold all but 20 acres at a profit of $30 per acre over the original cost per acre, the company regained the entire cost of the parcel. How many acres were originally purchased?

4. (4 points) Find the domain of $g(t) = \dfrac{\sqrt{8 - 5t}}{t^2 - 3t - 54}$.

5. (3 points) Find the missing terms which fit the pattern.

$$5, 12, 26, \underline{\quad}, \underline{\quad}, \underline{\quad}, 446$$

6. (5 points) Find the missing terms which fit the pattern, write in summation notation, and evaluate the sum

$$-54 - 33 - 12 + 9 + 30 + 51 + \underline{\quad} + \underline{\quad} + \underline{\quad} + 135$$

7. (5 points) Consider the recursive sequence given by $a_1 = 4$ and $a_{k+1} = \dfrac{12}{a_k - 2}$ for $k \geq 1$. Find a_6.

8. (2 points) Evaluate the ninety-ninth (99th) term of the sequence $(a_k)_{k=1}^{\infty} = \left(\dfrac{1}{k} \sum_{i=1}^{k} 3i - 7 \right)_{k=1}^{\infty}$.

9. Evaluate, if possible,

    (a) (3 points) $\displaystyle\sum_{k=1}^{10} k^2\left(k - \frac{2}{k}\right)$

    (b) (3 points) $\displaystyle\sum_{n=1}^{\infty} -2\left(-\frac{8}{5}\right)^{n-1}$

    (c) (3 points) $\displaystyle\sum_{n=0}^{\infty} -2\left(-\frac{3}{5}\right)^{n}$

10. (5 points) A \$14,000 investment is made into an account that pays a nominal rate of r_n compounded annually. After twenty years, the amount has doubled. Find an expression for the exact nominal rate and give an approximation of this rate as a percentage to one decimal place.

11. (6 points) The present value of a 25-year mortgage is \$150,000. If the nominal interest rate is 3% compounded monthly, with the payments made at the end of each month, write the simplified expression for the total value of the payments over the 25-year mortgage. **You may use decimals in this answer, but you may not round your answer.**

12. (6 points) Write the 4×3 matrix, A, whose entries are given by $a_{ij} = i^2 - 3j$.

13. Consider the following matrices:

$$A = \begin{bmatrix} 5 & 1 \\ -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ -3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Evaluate, if possible, If not possible, give reason(s).

(a) (4 points) $2A - BB^T$

(b) (4 points) BAC

(c) (4 points) $AB - (CA)^T$

(d) (4 points) $A^2 - 8A + 17I$

14. (3 points) A matrix M is called a *Jones matrix* if $M^2 = M + I$. Prove that if A is a Jones matrix, then so is A^T.

15. (6 points) An equilibrium occurs when supply equals demand. If the supply and demand equations of a certain product are $120p - q - 240 = 0$ and $100p + q - 1200 = 0$ respectively, find the equilibrium price and quantity.

16. (2 points) The difference in price of two items before a 15% sales tax is imposed is $6. What is the difference in price after the sales tax is imposed? Give your answer to the nearest penny.

17. (4 points) Assume the augmented matrix for the system of equations

$$\begin{cases} x_1 + 3x_2 - 2x_3 + x_4 = -2 \\ 2x_1 + 7x_2 + x_3 - 2x_4 = 3 \\ -x_1 - x_2 + 12x_3 = -6 \end{cases}$$

reduces to

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 30 & -74 \\ 0 & 1 & 0 & -9 & 22 \\ 0 & 0 & 1 & 1 & -3 \end{array}\right].$$

Write the solution of the system of equations.

18. (8 points) Solve the following system of equations using matrix reduction.

$$\begin{cases} x + z = 2 \\ y + z = -4 \\ x + y - z = -5 \end{cases}$$