Bishop's University

Math 196: Final Exam
Fall 2017

Name:	
Student #:	
Time:	180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

- All answers must be exact (no decimals allowed) unless specifically directed otherwise.

- All dollar values must be given to the nearest penny, unless otherwise indicated.

- Do not remove any pages from this test.

- The back of each page may be used for scrap paper.

- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other electronic calculators are permitted.

Useful Formulas

$$A = P\left(1 + \frac{r_n}{m}\right)^{mt}$$

$$r_e = \left(1 + \frac{r_n}{m}\right)^{m} - 1$$

$$S = R\left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt} - 1}{\frac{r_n}{m}}\right)$$

$$P = R\left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{-mt}}{\frac{r_n}{m}}\right)$$
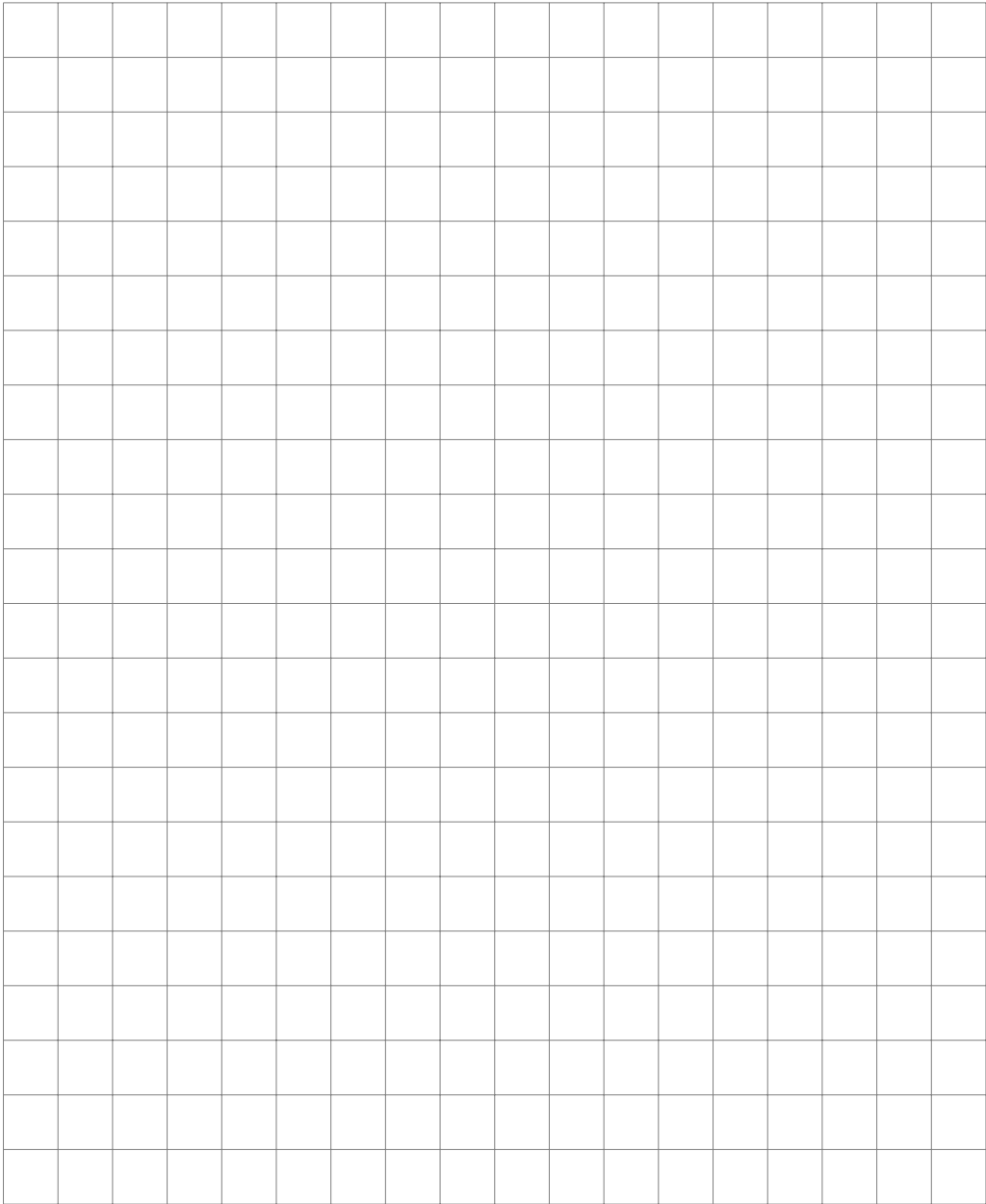
$$S = R\left(\frac{\left(1 + \frac{r_n}{m}\right)^{mt+1} - 1}{\frac{r_n}{m}} - 1\right)$$

$$P = R\left(\frac{1 - \left(1 + \frac{r_n}{m}\right)^{1-mt}}{\frac{r_n}{m}} + 1\right)$$

Page	Points	Score
2	15	
3	15	
4	15	
5	20	
6	20	
7	15	
Total:	100	

1. (10 points) Sketch the region described by these inequalities, remembering to shade the **EXCLUDED** regions. Proper scaling and placement of axes will be taken into account in the grading.

$$\begin{cases} 3x - 6y & \leq & 12 \\ 2x - 3y & \geq & -6 \\ 7x + 4y & \leq & 28 \\ 5x - 3y & \geq & -15 \end{cases}$$

2. (5 points) Solve the following inequality, $\left| \dfrac{3}{4}x - 4 \right| > \dfrac{3}{17}$, and write the solution using interval notation.

3. Evaluate, if possible,

(a) (5 points) $\displaystyle\sum_{k=1}^{40} k^2 + 3k - 4$

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{7}{10}\right)^{n+2}$

4. (5 points) Consider the recursive sequence given by $a_1 = 55$, $a_2 = 34$, and $a_k = a_{k-2} - a_{k-1}$ for $k \geq 3$. Find a_7.

5. A 25-year mortgage for \$160,000 is granted to a new homeowner. The nominal interest rate is 3.12%, compounded monthly, with the payments made at the beginning of each compounding period.

  (a) (5 points) Compute the regular monthly payment.

  (b) (10 points) After ten years of making regular monthly payments, the homeowner decides to move to weekly payments. Assuming the interest is then compounded weekly, and the payments are made at the beginning of each week, does the homeowner save any money over the life of the mortgage (25 years)? Justify your answer and give the value of the savings (or extra cost) by making the switch.

6. Let

$$A = \begin{bmatrix} 4 & -1 & 2 \\ 0 & 6 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ -5 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -7 & 3 \\ 12 & -14 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Evaluate, if possible, the following. If not possible, give reasons.

(a) (5 points) $B^T A^T - C + 7I$.

(b) (5 points) $D(B^T + A)$.

7. (10 points) A portfolio manager has two stock portfolios have different rates of growth. Portfolio Safe has a 5% growth rate, and Portfolio Risky has a 12% growth rate. How many share of each Portfolio should an investor purchase if she wishes to have 2000 shares with an overall growth rate 7.8%?

8. (5 points) A three character password must have one lower-case letter, one upper-case letter, and one number. Assuming the English alphabet, and the numbers 0 to 9, how many different password are possible?

9. (5 points) How many distinguishable arrangements of all the letters in the word ILLEGIBLE are possible?

10. (5 points) A jelly bean is selected randomly from a bag which contains five red, nine white, and two green jelly beans. Find the probability that the jelly bean is not white.

11. (5 points) A fair coin is tossed five times in a row. Find the probability of getting exactly three heads if the third toss is a head.

12. At the beginning of a tournament, a certain curling team has a 72% probability of winning the first game. If the team win the first game, they have an 87% probability of winning the next game. If the team does not win the first game, they have a 51% probability of losing the next game. No ties are possible. **For this question, the answers are to be decimal numbers with four decimal places.**

(a) (8 points) Find the indicated probabilities. You may use the probability tree given below.

W2 $P(W1 \cap W2) =$

W1

L2 $P(W1 \cap L2) =$

W2 $P(L1 \cap W2) =$

L1

L2 $P(L1 \cap L2) =$
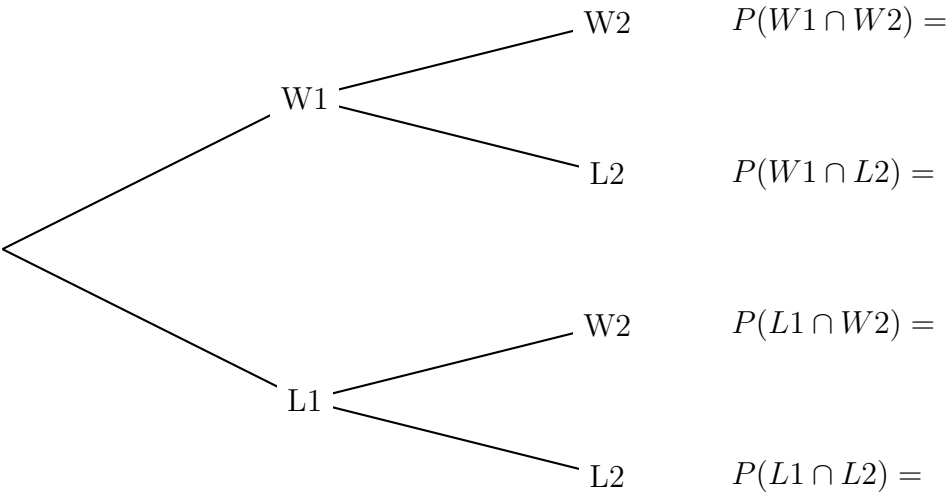
(b) (3 points) Find the probability that the team will win the second game.

(c) (2 points) If the team won the second game, find the probability that they lost the first game.

(d) (2 points) If the team lost the second game, find the probability that they won the first game.