Bishop's University

Math 196: Final Exam
Fall 2018

Name: ______________________________

Student #: ______________________________

Time: 180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- All dollar values must be given to the nearest penny, unless otherwise indicated.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Useful Formulas

$$A = P\left(1+\frac{r_n}{m}\right)^{mt}$$

$$r_e = \left(1+\frac{r_n}{m}\right)^{m} - 1$$

$$S = R\left(\frac{\left(1+\frac{r_n}{m}\right)^{mt}-1}{\frac{r_n}{m}}\right)$$

$$P = R\left(\frac{1-\left(1+\frac{r_n}{m}\right)^{-mt}}{\frac{r_n}{m}}\right)$$

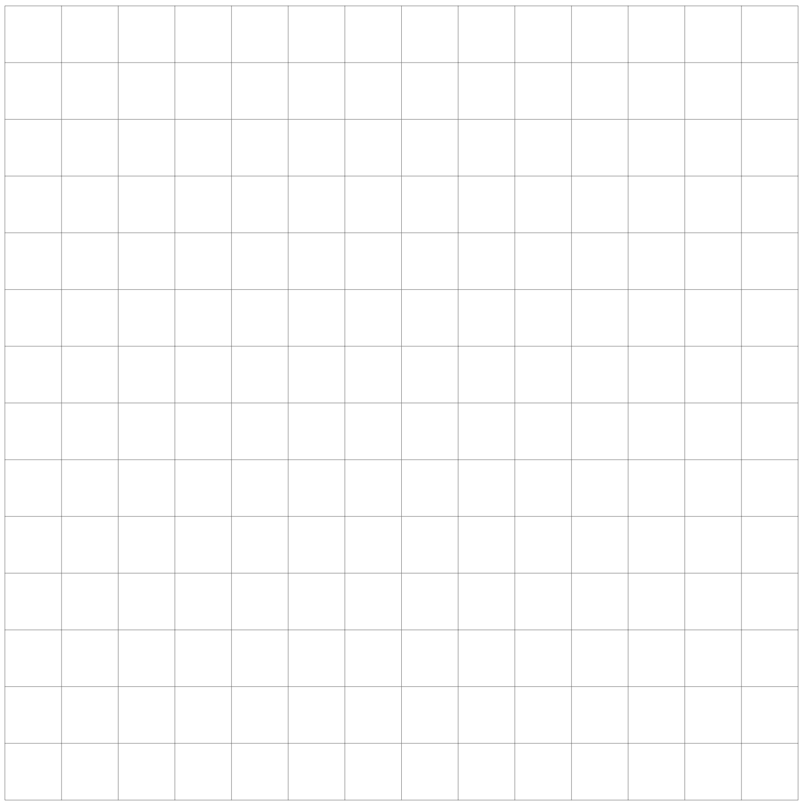
$$S = R\left(\frac{\left(1+\frac{r_n}{m}\right)^{mt+1}-1}{\frac{r_n}{m}}-1\right)$$

$$P = R\left(\frac{1-\left(1+\frac{r_n}{m}\right)^{1-mt}}{\frac{r_n}{m}}+1\right)$$

Page	Points	Score
2	15	
3	15	
4	15	
5	15	
6	20	
7	20	
Total:	100	

1. (10 points) Sketch the region described by these inequalities, remembering to shade the **EXCLUDED** regions. Proper scaling and placement of axes will be taken into account in the grading.

$$\begin{cases} 2x + 3y & \leq & 6 \\ x - y & \geq & -2 \\ 2x + y & \geq & -2 \\ 3x - y & \leq & 3 \end{cases}$$



2. (5 points) Let the intervals $(-\infty, -14] \cup [6, \infty)$ be the solution of an absolute value inequality. Write the absolute value inequality.

3. Evaluate, if possible,

   (a) (3 points) $\sum_{i=9}^{35} 6$

   (b) (3 points) $\sum_{k=1}^{40} k^2 + 3k - 4$

   (c) (4 points) $\sum_{n=1}^{\infty} 5\left(\frac{7}{10}\right)^{n-1}$

4. (5 points) Consider the recursive sequence given by $a_1 = 8$, $a_2 = 2$, and $a_k = \frac{a_{k-2}}{a_{k-1}}$ for $k \geq 3$. Find a_7.

5. (3 points) Find the effective rate that is equivalent to the nominal rate of 5.85%, compounded weekly. **Give the answer as a percentage with two decimal places.**

6. (3 points) What is the initial investment which give \$12,675 after 13 years at a nominal rate of 4.27%, compounded daily?

7. (5 points) A 7-year car loan for \$35,000 is granted to a new car owner. The nominal interest rate is 3.6%, compounded monthly, with the payments made at the end of each month. How much is left owing on the loan after 6 years? **Hint: Compute the present value of the annuity for the remaining years.**

8. (4 points) Use the rule of thumb given in class to estimate how long it will take an investment of \$1000 to grow to \$4000 if the annual effective rate is 24%. Then find the exact number of years **(to 3 decimal places)** required to reach the stated goal.

9. Let

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Evaluate, if possible, the following. If not possible, give reasons.

(a) (5 points) $AB + 5C - 7I$.

(b) (5 points) $(B + A^T)D^T$.

10. (5 points) Find values x and y such that

$$x \begin{bmatrix} 3 \\ 2 \end{bmatrix} + y \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

11. (5 points) Samantha has a bag of filled with chocolates. Ten of the chocolates are mint flavoured, the others are strawberry flavoured. Samantha takes one chocolate at random from the bag and eats it. She then takes another chocolate at random from the bag and eats that one as well. If the probability that Samantha chose two mint flavoured chocolates is $\frac{3}{14}$, find the number of chocolates that were originally in the bag.

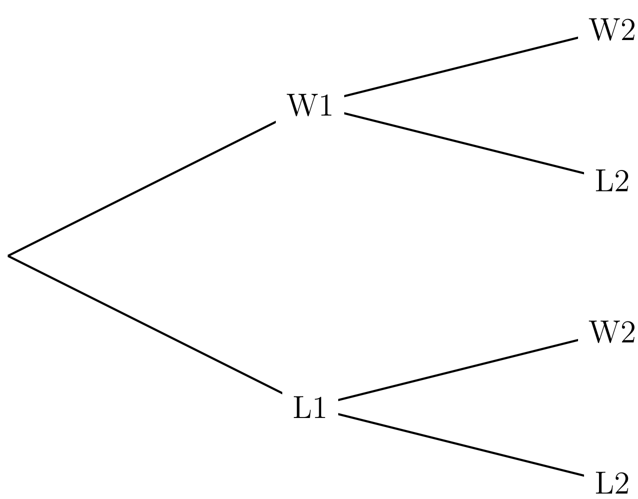
12. (5 points) How many distinguishable arrangements of all the letters in the word BOOKKEEPER are possible?

13. (5 points) Two jelly beans are selected randomly, without replacement, from a bag which contains five red, nine white, and two green jelly beans. Find the probability that both jelly beans are not white.

14. (5 points) A fair coin is tossed five times in a row. Find the probability of getting exactly two heads if the second toss is a tail.

15. At the beginning of a tournament, a certain curling team has a 73% probability of losing the first game. If the team win the first game, they have an 53% probability of winning the next game. If the team does not win the first game, they have a 87% probability of losing the next game. No ties are possible. **For this question, the answers are to be decimal numbers with four decimal places.**

(a) (8 points) Find the indicated probabilities. You may use the probability tree given below if it is helpful.



$P(W1 \cap W2) =$

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(b) (4 points) Find the probability that the team will lose the second game.

(c) (4 points) If the team won the second game, find the probability that they won the first game.

(d) (4 points) If the team lost the second game, find the probability that they lost the first game.