Bishop's University

Math 197: Final Exam
Winter 2013

Name: _____

Student #: _____

Time: 3 hours

- This exam is 180 minutes in length.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

Page	Points	Score
2	20	
3	11	
4	10	
5	8	
6	16	
7	10	
8	8	
9	8	
10	8	
Total:	99	

1. Differentiate, and DO NOT SIMPLIFY

(a) (4 points) $y = 5x^3 + 3x - 7x^{-2}$

(b) (4 points) $f(x) = x^2 \ln 3x$

(c) (4 points) $g(t) = \dfrac{e^{-t}}{e^{3t} + 1}$

(d) (4 points) $w = 2^x$

(e) (4 points) $z(r) = (3r^2 + 5s^2 r)^3$ where s is a constant.

2. Solve for x.

   (a) (2 points) $4^{3x-2} = 64$

   (b) (2 points) $\ln x = \ln 12 - \ln 2 - \ln(x-1)$

3. Consider the implicit function $(x^2 + y^2) + x(x^2 + 3y^3) = 40$

   (a) (4 points) Find $\dfrac{dy}{dx}$

   (b) (3 points) Find the equation of the tangent line to the curve at $(2, 2)$.

4. (6 points) Let $f(x) = e^{x^2}$. Find and simplify $f^{(3)}(x)$.

5. (4 points) Suppose that the total cost for a manufacturer is given by

$$c = \frac{4q^2}{\sqrt{q^2 + 21}} + 600.$$

Which is larger, the marginal cost or the average cost, when $q = 10$, and by how much?

6. (8 points) Given that

$$f(x) = 2x + \frac{5}{2}\ln\left|\frac{x+2}{x-2}\right|, \qquad f'(x) = \frac{2x^2-18}{x^2-4}, \qquad \text{and} \qquad f''(x) = \frac{20x}{(x^2-4)^2}$$

Find the intervals of increase and decrease, and determine the intervals of concavity.

7. Find the indicate partial derivative(s), and DO NOT SIMPLIFY

(a) (4 points) $f(x,y) = x^3y^2 - x(3x-2y)^2$; $f_x(x,y)$

(b) (4 points) $z = \dfrac{3x^2+y^2}{x^2+3y^2}$; $\dfrac{\partial z}{\partial y}$

(c) (4 points) $f(x,y,z) = (3x+y)(4y-2z)(5z+3x)$; f_z

(d) (4 points) $h(r,s) = (3r^2+5s^2r)^3$; h_r and h_s

8. (6 points) Let $f(x, y, z) = \dfrac{3xy^2}{z} + x^3yz$. Find all second order partial derivatives.

9. (4 points) Let $z = (x^2y - 3xy^2)^2$ and let $x = r^2s$ and $y = \dfrac{s^2}{r^2 + 1}$. Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and evaluate $\left.\dfrac{\partial z}{\partial s}\right|_{r=1,s=2}$.

10. (8 points) A farmer wishes to fence in 60,000 m^2 of land in a rectangular field along a straight road. The fencing he plans to use along the road cost \$10/m and the fencing along the other three sides costs \$5/m. Find the dimensions of the field which minimize the cost of the fence.

11. (8 points) Find and classify the critical points of $f(x, y) = x^4 + y^4 - 4xy + 4$.

12. (8 points) A company sells two competetive products, A and B, for which the demand equations are

$$p_A = 35 - 2q_A^2 + q_B, \quad \text{and} \quad p_B = 20 - q_B + q_A.$$

The joint-cost function is

$$c = 8 - 2q_A^3 + 3q_Aq_B + 30q_A + 12q_B + \frac{1}{2}q_A^2.$$

How many units of A and B should be sold to obtain a relative maximum profit for the company?