Bishop's University

Math 197: Final Exam
Winter 2014

Last Name: _____

First Name(s): _____

Student #: _____

Time: 180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- All dollar values must be given to the nearest penny, unless otherwise indicated.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.

Page	Points	Score
2	20	
3	9	
4	15	
5	12	
6	16	
7	5	
8	8	
9	8	
10	8	
Total:	101	

1. Differentiate, and DO NOT SIMPLIFY

(a) (4 points) $y = 7x^2 - 15x + 5^3 - 7x^{-1}$

(b) (4 points) $f(x) = x^2 e^{2x}$

(c) (4 points) $g(t) = \dfrac{\ln(t^2+1)}{3t^4+1}$

(d) (4 points) $w = \log_7 x$

(e) (4 points) $z(s) = \dfrac{1}{\sqrt{1-s^2}}$

2. Solve for x.

(a) (3 points) $15(3^x + 6) = 95$

(b) (3 points) $\ln(x - 1) = \ln 12 - \ln x$

(c) (3 points) $4^x - 6 \cdot 2^x + 8 = 0$

3. Consider the implicit function $x^2 + 2y^2 = (xy)^2$

   (a) (4 points) Find $\dfrac{dy}{dx}$

   (b) (3 points) Find the equation of the tangent line to the curve at the point $\left(\dfrac{3}{2}, 3\right)$.

   (c) (2 points) Is the graph increasing or decreasing near $\left(\dfrac{3}{2}, 3\right)$? Justify your answer.

4. (6 points) Let $f(x) = \ln(1 - x^2)$. Find and simplify $f'''(x)$.

5. (4 points) Use logarithmic differentiation to differentiate $y = \dfrac{(x^2-16)(x^2+4)^2}{e^x}$.

6. (8 points) Given that

$$f(x) = xe^{-\frac{x^2}{2}}, \qquad f'(x) = (1-x^2)e^{-\frac{x^2}{2}}, \qquad \text{and} \qquad f''(x) = (x^3-3x)e^{-\frac{x^2}{2}}$$

Determine the intervals of concavity, and use the second derivative test to classify the critical points.

7. Find the indicated partial derivative(s), and DO NOT SIMPLIFY

(a) (4 points) $f(x,y) = (x^2+y^2-4)^3 - 108y^2$; $f_y(x,y)$

(b) (4 points) $z = \ln((3x^2+y^2)(x^2+3y^2))$; $\dfrac{\partial z}{\partial x}$

(c) (4 points) $f(x,y,z) = (3y+z)(4z-2x)(5x+3y)$; f_y

(d) (4 points) $h(r,s) = \left(\dfrac{r^4s^4}{r^2+1} - \dfrac{3r^2s^4}{(r^2+1)^2}\right)^2$; h_s

8. (5 points) Let $f(x, y, z) = \dfrac{3x^2}{y} + xz^3$. Find all second order partial derivatives.

9. (8 points) Suppose that the total cost, in thousands of dollars, for a manufacturer is given by

$$C(q) = \frac{1}{6}q^3 - 18q^2 + 15q + 500,$$

where q is the quantity of units produced, measured in hundreds. How many units must be produced to minimize **marginal cost**?

10. (8 points) Find and classify the critical points of $f(x, y) = x^3 + 8y^3 - 3x^2 + 6y^2 + 7$.

11. (8 points) A company manufactures open-top crates. The most popular model has a volume of 4 cubic feet. If the material costs \$3 per square foot, find the dimensions of the crate which minimizes the cost.