Bishop's University

Math 197: Final Exam
Winter 2015

Last Name: _____

First Name(s): _____

Student #: _____

Time: 180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.

× \ +	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	0	2	3	4	5	6	7	8	9	10	11
2	0	2	4	5	6	7	8	9	10	11	12
3	0	3	6	6	7	8	9	10	11	12	13
4	0	4	8	12	8	9	10	11	12	13	14
5	0	5	10	15	20	10	11	12	13	14	15
6	0	6	12	18	24	30	12	13	14	15	16
7	0	7	14	21	28	35	42	14	15	16	17
8	0	8	16	24	32	40	48	56	16	17	18
9	0	9	18	27	36	45	54	63	72	18	19
10	0	10	20	30	40	50	60	70	80	90	20

Page	Points	Score
2	20	
3	11	
4	10	
5	10	
6	16	
7	11	
8	8	
9	8	
10	13	
Total:	107	

1. Differentiate, and DO NOT SIMPLIFY

   (a) (4 points) $y = 2x^7 - 3x^5 + 7x^{-2} - 15\sqrt[3]{x}$

   (b) (4 points) $f(x) = e^x \ln x$

   (c) (4 points) $w = 7^x$

   (d) (4 points) $z(s) = (3r^2 + 5s^2r)^3$ where r is a constant.

2. (4 points) Differentiate and simplify $g(t) = \dfrac{\ln(t^2+1)}{1+\ln(t^2+1)}$.

3. (4 points) Use the limit definition of the derivative to find $f'(x)$ when $f(x) = 2x^2 + 3x - 7$.

4. Consider the function $y = (\sqrt{x})^{\sqrt{x}}$

   (a) (4 points) Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

   (b) (3 points) Find the equation of the tangent line to the curve at $x = e^2$.

5. (6 points) Let $f(x) = e^{-\frac{1}{2}x^2}$. Find and simplify $\dfrac{d^3f}{dx^3}$.

6. (4 points) Suppose that the total cost for a manufacturer is given by

$$C = \frac{4q^2}{\sqrt{q^2 + 7500}} + 200.$$

What is the rate of change of the average cost when $q = 50$?

7. (10 points) Given that

$$f(x) = \frac{x}{x^2+3}, \qquad f'(x) = \frac{3-x^2}{(x^2+3)^2}, \qquad \text{and} \qquad f''(x) = \frac{2x(x^2-9)}{(x^2+3)^3}$$

Find the intervals of increase and decrease, and determine the intervals of concavity. Determine the where the relative extrema occur.

8. Find the indicated partial derivative(s), and DO NOT SIMPLIFY

(a) (4 points) $f(x,y) = x^3y^2 - x(3x-2y)^2$; $f_x(x,y)$

(b) (4 points) $z = \dfrac{3x^2+y^2}{x^2+3y^2}$; $\dfrac{\partial z}{\partial y}$

(c) (4 points) $f(x,y,z) = \dfrac{3x^2y}{z} + xyz^3$; f_y

(d) (4 points) $h(r,s) = (3r^2+5s^2r)^3$; h_{rs}

9. (6 points) Let $f(x, y, z) = (3y + z)(4z - 2x)(5x + 3y)$. Find all second order partial derivatives.

10. (5 points) Given the demand equation $p = 22 - \sqrt{q^2 + 29}$, find the elasticity of demand at $q = 14$, and determine whether demand is elastic, inelastic, or has unit elasticity. Given this information, how should the quantity be change to increase revenue, if possible?

11. (8 points) A farmer wishes to spend \$12,000 in fencing a region of his property which borders a straight road. The fencing he plans to use along the road cost \$10 per foot and the fencing along the other three sides costs \$6 per foot. Find the dimensions of the field which maximize the area of the fenced region.

12. (8 points) Use the method of Lagrange multipliers to find the maximum of $f(x, y) = \frac{1}{2}x^2 - 2y^2 + 7$ subject to the restriction that $xy = 2$.

13. (8 points) Classify the critical points of $f(p, q) = p^6 + q^6 + 6pq$.

14. (5 points) Suppose that $y'' = 120x^3 - 72x^2 + 6$, and $y(0) = 1$ and $y(1) = 0$. Find y.