Bishop's University

Math 197: Final Exam
Winter 2016

Last Name: _____

First Name(s): _____

Student #: _____

Time: 180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.

Page	Points	Score
2	25	
3	15	
4	20	
5	15	
6	20	
7	15	
8	17	
9	18	
Total:	145	

1. Differentiate, and DO NOT SIMPLIFY

   (a) (5 points) $y = 5x^4 - \dfrac{7}{x^3} + e^2$

   (b) (5 points) $f(x) = x\ln(x^2+1)$

   (c) (5 points) $w = 4^u \log_4\left(\dfrac{1}{u}\right)$

   (d) (5 points) $z(s) = \dfrac{4rs - rs^3}{r^2 + 2rs + s^2}$ where r is a constant.

2. (5 points) Differentiate and simplify $g(x) = \dfrac{\sqrt{x^2-1}}{\sqrt{x^2+1}}$.

3. (10 points) Use the limit definition of the derivative to find $f'(x)$ when $f(x) = 6x^2 - 5x + 4$.

4. (5 points) For what values of a and b is the function

$$f(x) = \begin{cases} x^2 + 2x - 2 & x < -2 \\ ax^2 + bx & -2 \leq x < 3 \\ 9x - x^2 & x \geq 3 \end{cases}$$

continuous?

5. Consider the function $y = (x^2 + 1)^x$

   (a) (6 points) Use logarithmic differentiation to find $\dfrac{dy}{dx}$.

   (b) (4 points) Find the equation of the tangent line to the curve at $x = 2$.

6. (10 points) Let $f(x) = e^{x^2}$. Find and simplify $\dfrac{d^3 f}{dx^3}$.

7. (5 points) Suppose that the total cost for a manufacturer is given by

$$C = \frac{3q}{\sqrt{q^2 + 900}} + 500q + 1000.$$

What is the relative rate of change of the marginal cost when $q = 40$?

8. (10 points) Given that

$$f(x) = \frac{2x^2 + 1}{x^2 + 1}, \qquad f'(x) = \frac{2x}{(x^2 + 1)^2}, \qquad \text{and} \qquad f''(x) = -\frac{2(3x^2 - 1)}{(x^2 + 1)^3}$$

Find the intervals of increase and decrease, and determine the intervals of concavity. Determine the where the relative extrema occur.

9. Find the indicated partial derivative(s), and DO NOT SIMPLIFY

(a) (5 points) $f(x,y) = x^3 + 3x^2y - 3xy^2 + y^3$; $f_x(x,y)$

(b) (5 points) $z = \dfrac{3x + y^2}{x^2 + 3y}$; $\dfrac{\partial z}{\partial y}$

(c) (5 points) $f(x,y,z) = e^{xy^2z^3} \ln(x^3y^2z)$; f_y

10. (5 points) Let $h(r,s) = \dfrac{4rs - rs^3}{r^2 + 2rs + s^2}$. Evaluate $h_{rs}(2,1)$

11. (6 points) Let $f(x,y) = \dfrac{4y-3x}{5x+3y}$. Find all second order partial derivatives.

12. (9 points) Given the demand equation $p = 25 + \dfrac{6250}{q^3}$, use elasticity of demand to determine the quantity to sell to achieve maximum revenue. What is the maximum revenue?

13. (10 points) Use the method of Lagrange multipliers to find the maximum of $f(x, y) = \frac{1}{2}x^2 + 2y^2$ subject to the restriction $x^2 + 2x + y^2 = 0$.

14. (7 points) Find the absolute extrema of $f(x, y) = \frac{1}{2}x^2 + 2y^2$ subject to the restriction $x^2 + 2x + y^2 = 0$ in the following manner: Substitute $y^2 = -x^2 - 2x$ into the function f to create a function of a single variable. Also note that it is true that the values of x are in the interval $[-2, 0]$.

15. (10 points) Classify the critical points of $f(p, q) = p^3 + q^3 - 3pq$.

16. (8 points) Find a function F such that $F'' = 4(3x - 2)^2$, and $F'(1) = 1$ and $F(1) = 0$.