Bishop's University

Math 197: Final Exam
Winter 2017

Last Name: ______________________

First Name(s): ______________________

Student #: ______________________

Time: 180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper. No additional scrap paper allowed.
- A **Casio fx260-solar** calculator is permitted.

Page	Points	Score
2	30	
3	30	
4	30	
5	15	
6	20	
7	10	
8	10	
9	15	
Total:	160	

1. Evaluate the limit, if it exists. If the limit is infinite, specify whether it is positive or negative infinity. If the limit does not exist, explain why.

(a) (5 points) $\lim_{x\to 8} 2x^3 - 6x + 7$

(b) (5 points) $\lim_{x\to -4} \sqrt{25}$

(c) (5 points) $\lim_{t\to 3} \frac{6t}{8 - t^2}$

(d) (5 points) $\lim_{w\to 1} \frac{w^4 - 1}{w^2 - 1}$

(e) (5 points) $\lim_{k\to 0^-} k\left(8 + \frac{6}{k}\right)$

(f) (5 points) $\lim_{x\to\infty} \frac{(6-x)(5+5x)}{(3-8x)(9+10x)}$

2. Differentiate, and DO NOT SIMPLIFY

(a) (5 points) $y = 4x^3 - \dfrac{7}{\sqrt{x^3}} + e^{-1}$

(b) (5 points) $f(x) = x\ln(1 - x^2)$

(c) (5 points) $w = \dfrac{e^x}{\ln x}$

(d) (5 points) $z(s) = (4rs - rs^3)(r^2 + 2rs + s^2)$ where r is a constant.

(e) (5 points) $p = \dfrac{(3q-2)^4(5-4q)^2}{7q^2 - 3q + 5}$

(f) (5 points) $y = 5^x$

3. Evaluate the following integrals.

   (a) (5 points) $\int 5\sqrt{e}dx$

   (b) (5 points) $\int_{-1}^{2} x^2 dx$

   (c) (5 points) $\int (4x^3 - \frac{7}{\sqrt{x^3}} + e^{-1})dx$

   (d) (5 points) $\int t\sqrt{1-t^2}dt$

   (e) (5 points) $\int_0^1 (w+1)(2w-3)dw$

   (f) (5 points) $\int_{-5}^{5} (8x^5 - 13x^3 + 7x)dx$

4. (5 points) For what values of a and b is the function

$$f(x) = \begin{cases} x^2 + 2x - 2 & x < -1 \\ ax + b & -1 \le x < 2 \\ 9x - x^2 & x \ge 2 \end{cases}$$

continuous?

5. (10 points) Let $f(x) = x^2 e^x$. Find and simplify $\dfrac{d^3 f}{dx^3}$.

6. (10 points) A farmer wishes to spend \$8,000 in fencing a rectangular region of his property which borders a straight road. The fencing he plans to use along the road cost \$12 per foot and the fencing along the other three sides costs \$8 foot. Find the dimensions of the field which maximize the area of the fenced region.

7. (10 points) Verify that $p(x) = \frac{3}{8}x - \frac{3}{32}x^2$ is a probability density function on the interval $[0, 4]$, and find the probability that $2 \leq x \leq 3$.

8. (5 points) Use the limit definition of the derivative to find $f'(x)$ when $f(x) = x^2 + x$.

9. (5 points) Find the equation of the tangent line to $(3x+y)^2+(2x+y)^2 = 2y+x^3$ at the point $(-2, 5)$.

10. (5 points) Use logarithmic differentiation to find y' where

$$y = \sqrt[3]{\frac{(3x+2)^4(2x^2-x+1)^2}{(4-7x^3)^5(e^{x^2+1})}}.$$

11. (5 points) Given the demand equation $p = 54 - \sqrt{q^2+17}$, find the elasticity of demand at $q = 8$, and determine whether demand is elastic, inelastic, or has unit elasticity. Given this information, how should the price be change to increase revenue, if possible?

12. (5 points) Suppose that the total cost for a manufacturer is given by

$$C = \frac{3q}{q^2 + 900} + 500q + 1000.$$

What is the relative rate of change of the average cost when $q = 40$?

13. (10 points) Given that

$$f(x) = \frac{e^x}{x}, \qquad f'(x) = \frac{e^x(x-1)}{x^2}, \qquad \text{and} \qquad f''(x) = \frac{e^x(x^2 - 2x + 2)}{x^3}$$

Determine the intervals of concavity and state which types of relative extrema occur and where they occur. Is there an absolute extremum (why or why not)?