Bishop's University

Math 197: Final Exam
Winter 2019

Last Name: _____

First Name(s): _____

Student #: _____

Time: 180 minutes

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper. No additional scrap paper allowed.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted.
- **If you are found to be in possession of any unauthorized material (ie cell phone, incorrect calculator, notes, etc.), you have committed an academic offense and will receive a grade of 0 on the final exam.**

Page	Points	Score
2	30	
3	30	
4	30	
5	10	
6	15	
7	10	
8	15	
Total:	140	

1. Evaluate the limit, if it exists. If the limit is infinite, specify whether it is positive or negative infinity. If the limit does not exist, explain why.

   (a) (5 points) $\lim_{x \to 5} 3x^3 - 4x + 7$

   (b) (5 points) $\lim_{a \to -6} b$

   (c) (5 points) $\lim_{t \to -3^+} \dfrac{6t}{9 - t^2}$

   (d) (5 points) $\lim_{w \to -1} \dfrac{w^4 - 1}{w^2 - 1}$

   (e) (5 points) $\lim_{k \to 0} k^2 \left(4 + \dfrac{12}{k}\right)$

   (f) (5 points) $\lim_{x \to -\infty} \dfrac{(6 + 2x^3)(5 + 3x^2)}{(5 - 6x)(9 - 11x^4)}$

2. Differentiate, and DO NOT SIMPLIFY

(a) (5 points) $y = 3x^4 + \dfrac{11}{\sqrt[3]{x^2}} + \sqrt{17}$

(b) (5 points) $f(x) = x^2 \ln(1 + 3x)$

(c) (5 points) $w = \dfrac{e^x}{x}$

(d) (5 points) $z(s) = \left(4r^2 s - \dfrac{r}{s^3}\right)(r^2 + 3rs + 4s^2)$ where r is a constant.

(e) (5 points) $p = \dfrac{(3q-2)^4(5-4q)^2}{7q^2 - 3q + 5}$

(f) (5 points) $y = \log_{10} x$

3. Evaluate the following integrals.

(a) (5 points) $\int_0^{\ln 2} (5e)dx$

(b) (5 points) $\int_{10}^{-10} 2x^5 dx$

(c) (5 points) $\int \frac{4x^3 - 7x}{\sqrt{x^3}} dx$

(d) (5 points) $\int \frac{t}{1 + 2t^2} dt$

(e) (5 points) $\int_0^1 (2w + 1)(3w - 2)dw$

(f) (5 points) $\int_{-1}^1 (21x^6 + 10x^4 - 12x^2)dx$

4. (5 points) Let $f(x) = x^2 \ln(x)$. Find and simplify $\dfrac{d^3 f}{dx^3}$.

5. (5 points) A farmer wishes to enclose a rectangular region which measures 9,075 square feet on her property which borders a straight road for 200 feet. The fencing she plans to use along the road cost \$25 per foot and the fencing along the other three sides costs \$15 foot. Find the dimensions of the field which minimize the cost of the fencing.

6. (10 points) Verify that $p(x) = \dfrac{1}{x \ln 2}$ is a probability density function on the interval $[2, 4]$, and find the probability that $3 \leq x \leq 4$.

7. (5 points) Use the limit definition of the derivative to find $f'(x)$ when $f(x) = x^2 - 2x + 7$.

8. (5 points) Find the equation of the tangent line to $(x+3y)^3+(x+2y)^2 = y+3x^3-9$ at the point $(1,-1)$.

9. (5 points) Use logarithmic differentiation to find y' where

$$y = \sqrt{\frac{\sqrt[3]{(2x-3)(x^2-x)^3}}{(7-13x^2)^5\ln(x^2+1)}}.$$

10. (10 points) Given the demand equation $p = 8q - \sqrt{q^4 + 51}$, find the elasticity of demand at $q = 5$, and determine whether demand is elastic, inelastic, or has unit elasticity. A manager argues that the price should be decreased as that will gather more revenue through increased sales. Should the advice be followed? Briefly explain.

11. (5 points) Given that

$$f(x) = x^4 - 6x^2 + 5, \qquad f'(x) = 4x^3 - 12x, \qquad \text{and} \qquad f''(x) = 12x^2 - 12$$

Determine the intervals of concavity and state which types of relative extrema occur and where they occur. Is there an absolute extremum (why or why not)?