Bishop's University

Math 200: Final Exam
Fall 2021

Name: ______________________________

Student #: ______________________________

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**
- There are 5 bonus marks at the end of the exam. The maximum score possible will 105 points.

Page	Points	Score
2	15	
3	15	
4	10	
5	15	
6	20	
7	20	
8	10	
Total:	105	

1. (10 points) Show, via truth tables, that $P \to \sim Q$ and $\sim (P \wedge Q)$ are logically equivalent.

P	Q				
T	T				
T	F				
F	T				
F	F				

2. (5 points) Use the binomial theorem to find the coefficient of x^6y^3 in $(3x - 2y)^9$.

3. The Alumni of Saint Belousov University have decided to run a lottery in support of the University Tiddlywinks team. The lottery committee agree that ticket buyers will pick 5 letters from the 13 different letters that make up the name of the University: s, a, i, n, t, b, e, l, o, u, v, r, y.

   How many different tickets are possible if:

   (a) (5 points) The order in which the letters are picked is not important and no letter may be repeated (eg. *saint* and *stain* give the same ticket.)

   (b) (5 points) The order is taken into account and letters may be repeated (eg. *abbab* and *aabbb* are different tickets)

   (c) (5 points) The order is taken into account and letters may be repeated but exactly three of the letters must be vowels. (eg. *asalo*. Note: y is not considered to be a vowel in this instance.)

4. Use Mathematical Induction to prove:

   (a) (10 points) $\forall n \in \mathbb{N}, \;\; 2 + 5 + 8 + \cdots + (3n - 1) = \dfrac{n(3n+1)}{2}$

(b) (15 points) $\forall n \in \mathbb{N},\ \forall q > 0,\ \ (1+q)^n \geq 1 + nq$. In addition, show that the statement is not true if "$q > 0$" is replaced by "$q \in \mathbb{R}$".

5. (10 points) Let x and y be integers. Prove that if x^2y is even, then x is even or y is even.

6. (10 points) Prove that $\sqrt{7}$ is irrational. (You may assume that if p is prime and $p|a^2$ where $a \in \mathbb{Z}$ then $p|a$.)

7. (10 points) Prove that $\{5x + 12y | x, y \in \mathbb{Z}\} = \mathbb{Z}$.

8. (10 points) Prove: If A, B, and C are sets then $A - (B \cap C) = (A - B) \cup (A - C)$.

9. (10 points) Prove or disprove: The inequality $2^x \geq x + 1$ is true for all positive real numbers x.

10. (5 Bonus points) Prove $\forall n \in \mathbb{N}$, $\dfrac{1}{1} + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{2^n - 1} + \dfrac{1}{2^n} \geq 1 + \dfrac{n}{2}$.