

BISHOP'S UNIVERSITY

MATH 200: FINAL EXAM

Fall 2021

Name:	
Student #:	

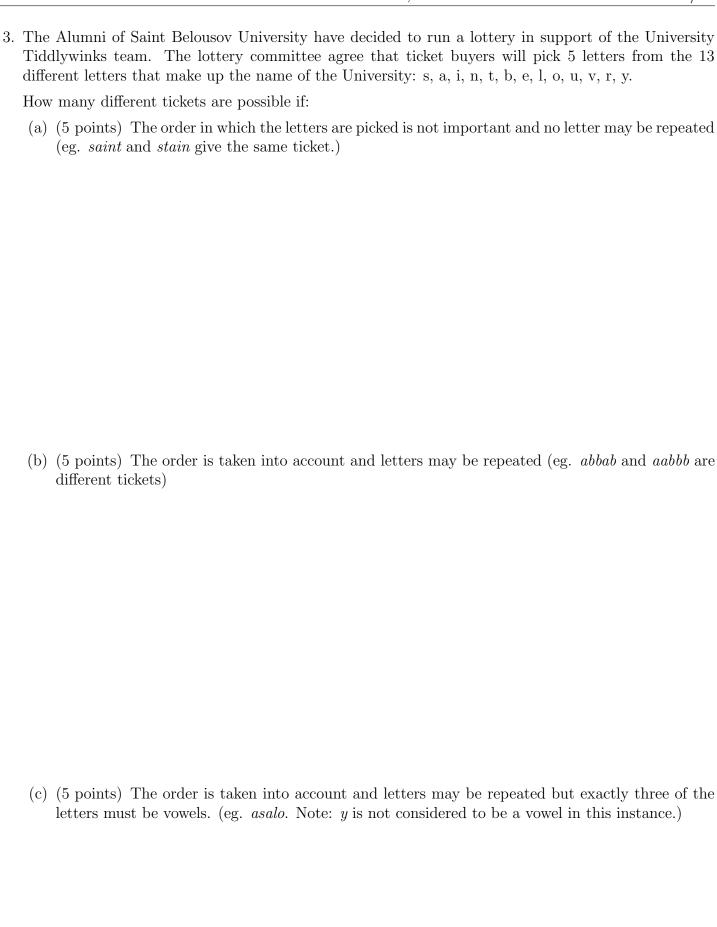
- Prepare neat solutions. Briefly justify your work, that is, make your reasoning clear.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A Casio fx260-solar or Casio fx260-solar II calculator is permitted. No other aids are permitted.
- Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.
- There are 5 bonus marks at the end of the exam. The maximum score possible will 105 points.

Page	Points	Score
2	15	
3	15	
4	10	
5	15	
6	20	
7	20	
8	10	
Total:	105	
8	10	

1. (10 points) Show, via truth tables, that $P \to \sim Q$ and $\sim (P \land Q)$ are logically equivalent.

P	Q		
Т	Т		
Т	F		
F	Т		
F	F		

2. (5 points) Use the binomial theorem to find the coefficient of x^6y^3 in $(3x-2y)^9$.



- 4. Use Mathematical Induction to prove:
 - (a) (10 points) $\forall n \in \mathbb{N}, \ 2+5+8+\cdots+(3n-1)=\frac{n(3n+1)}{2}$

(b) (15 points) $\forall n \in \mathbb{N}, \ \forall q > 0, \ (1+q)^n \ge 1+nq$. In addition, show that the statement is not true if "q > 0" is replaced by " $q \in \mathbb{R}$ ".

5. (10 points) Let x and y be integers. Prove that if x^2y is even, then x is even or y is even.

6. (10 points) Prove that $\sqrt{7}$ is irrational. (You may assume that if p is prime and $p|a^2$ where $a \in \mathbb{Z}$ then p|a.)

7. (10 points) Prove that $\{5x + 12y | x, y \in \mathbb{Z}\} = \mathbb{Z}$.

8. (10 points) Prove: If A, B, and C are sets then $A - (B \cap C) = (A - B) \cup (A - C)$.

9. (10 points) Prove or disprove: The inequality $2^x \ge x + 1$ is true for all positive real numbers x.

10. (5 Bonus points) Prove $\forall n \in \mathbb{N}, \ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^n - 1} + \frac{1}{2^n} \ge 1 + \frac{n}{2}.$