Bishop's University

Math 203: Final Exam
Fall 2015

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- This exam is 180 minutes in length.
- There is a total of 120 points on this exam.
- All solutions of congruences must be given in terms of the least positive residue.

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1. (a) (8 points) Use the Euclidean Algorithm to find $(2431, 1378)$.
   (b) (4 points) Find $[2431, 1378]$.
   (c) (8 points) Find integers x, y such that $2431x + 1378y = (2431, 1378)$.
2. (10 points) Find the digit in the hundreds place of 3^{3612}.
3. (10 points) Consider the relation R defined on $\mathbb{Z}$ as follows: aRb if and only if $7|a^2 - b^2$. Show that R is an equivalence relation.
4. (15 points) Solve $x^3 - x + 4 \equiv 0 \pmod{125}$
5. (15 points) Let $y(n)$ be the Dirichlet inverse of $\phi(n)$. Evaluate $y(36)$.
6. Use the fact that 2 is a primitive root (mod 13) to answer the following questions:
   (a) (6 points) Find all primitive roots (mod 13).
   (b) (7 points) Solve $x^5 \equiv 11 \pmod{13}$.
   (c) (7 points) Solve $3^x \equiv -4 \pmod{13}$.
7. (20 points) According to a well-known story, a Chinese general was fond of using modular arithmetic to count his troops. One day. he ordered the troops to line up in rows of 2, and found that there was one soldier left over at the end. Then he ordered the troops to line up in rows of 3, and again there was 1 soldier left over. Next he ordered the troops to line up in rows of 5, then rows of 7, and in each case there was exactly 1 soldier left over. Finally, when the troops were lined up in rows of 11, there were no soldiers left over, which pleased the general. Given that the general had no more that 15,000 troops, determine the largest number of soldiers in the general's army.

Choice questions (10 points): Do exactly one of the following questions

8. Let S_n be the sum of n consecutive odd natural numbers, not necessarily starting at 1.
   (a) If n is odd, prove $n|S_n$.
   (b) If p is prime, prove that p cannot be written as the sum of two or more consecutive odd natural numbers.
9. (a) For $n > 2$ prove that $2|\phi(n)$.
   (b) Find all n such that $\phi(\phi(n)) = 1$.