# Bishop's University

## Math 203: Final Exam
## Winter 2018

---

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- This exam is 180 minutes in length.
- There is a total of 100 points on this exam.
- All solutions of congruences must be given in terms of the least positive residue.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted.

---

1. (a) (5 points) Use the Euclidean Algorithm to find $(2431, 1378)$.
   (b) (5 points) Find integers $x, y$ such that $2431x + 1378y = (2431, 1378)$.
2. (a) (5 points) Find the prime power decomposition of the number $105,674,625$.
   (b) (5 points) Prove that if $n$ is composite, then $2^n - 1$ is composite.
3. (10 points) Find all solutions in the positive integers of $2x + 6y = 18$.
4. (7 points) Prove that every prime greater than 3 is congruent to either 1 or 5 (mod 6).
5. (3 points) What is the remainder when the number $4,765,796,538,056,749,753,790,623$ is divided by 11?
6. (10 points) Find $n$ such that $3^2|n$, $4^2|n+1$, and $5^2|n+2$.
7. (10 points) Find the digit in the hundreds place of $3^{3612}$.
8. (10 points) Calculate $d(n)$, $\sigma(n)$, and $\phi(n)$ for $n = 100116$.
9. (5 points) Are 1184 and 1210 an amicable pair?
10. (5 points) If $n$ is an even perfect number and $n > 6$, show that the sum of its digits is congruent to 1 (mod 9).
11. (5 points) Prove that if $(m, n) = 2$ then $\phi(mn) = 2\phi(m)\phi(n)$.
12. (5 points) Use the fact that 2 is a primitive root (mod 13) to find all primitive roots (mod 13).
13. (5 points) Verify that 2467 is prime and calculate $\left(\dfrac{1356}{2467}\right)$.
14. (5 points) Solve, if possible, $4x^2 - 7x + 12 \equiv 0 \pmod{61}$.