Bishop's University

Math 203: Final Exam
Winter 2021

Name: ______________________________

Student #: ______________________________

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- This exam is 180 minutes in length.
- All solutions of congruences must be given in terms of the least positive residue.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	5	
Total:	75	

1. (a) (5 points) Use the Euclidean Algorithm to find $(42823, 6409)$.

(b) (5 points) Find integers x, y such that $42823x + 6409y = (42823, 6409)$.

2. (5 points) Find the prime power decomposition of the number 72 576.

3. (5 points) Find, with justification, **the number of solutions** in the non-negative integers ($\{0, 1, 2, \ldots\}$) of $2x + 5y = 100$.

4. (5 points) A man bought a dozen pieces of fruit, apples and oranges, for 99 cents. If an apple costs 3 cents more than an orange, and he bought more apples than oranges, how many of each did he buy?

5. (5 points) What is the remainder when the number 585 987 448 204 883 847 382 293 085 463 is divided by 11?

6. (5 points) Find the digit in the tens place of 7^{4928}.

7. (5 points) Solve the system of congruences:

$$2x \equiv 1 \pmod{5}, \quad 3x \equiv 2 \pmod{7}, \quad 4x \equiv 3 \pmod{11}.$$

8. (5 points) Calculate $d(n)$, $\sigma(n)$, and $\phi(n)$ for $n = 10115$.

9. (5 points) Verify that 2819 is prime and calculate $\left(\frac{1156}{2819}\right)$.

10. (5 points) Prove that $(1 + 2 + \cdots + n) | n!$ if and only if $n + 1$ is composite.

11. (5 points) Prove that for an odd prime p, $2(p-3)! + 1 \equiv 0 \pmod{p}$

12. (5 points) A number n is called **super-perfect** if and only if $\sigma(\sigma(n)) = 2n$. Show that if $n = 2^k$, and $2^{k+1} - 1$ is prime, then n is super-perfect.

13. (5 points) Use the fact that 13 is a primitive root (mod 19) to find all primitive roots (mod 19).

14. (5 points) Solve, if possible, $3x^2 - 12x + 7 \equiv 0 \pmod{61}$.