Bishop's University

Math 203: Final Exam
Fall 2022

Name: ____________________

Student #: ____________________

- This exam is 180 minutes in length.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- All solutions of congruences must be given in terms of the least positive residue.
- You are permitted to use one (1) Authorized Memory Book and a Casio fx-260 Solar (II) calculator.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	15	
3	15	
4	15	
5	15	
6	15	
7	10	
8	15	
Total:	100	

1. (a) (7 points) Use the Euclidean Algorithm to find $(15732, 2772)$.

(b) (8 points) Find the solution of $15732x + 2772y = 108$. which has the smallest positive x value.

2. (8 points) Find the prime power decomposition of the number 30 492.

3. (7 points) What is the remainder when the number 448 204 382 293 085 is divided by 101? (**Write the number as sums of powers of 100. For example** $872\ 021 = 87 \times 100^2 + 20 \times 100 + 21$**.**)

4. (6 points) Find the digit in the tens place of 11^{2444}.

5. (9 points) Solve the system of congruences:

$$x \equiv 1 \pmod{9}, \quad x \equiv 4 \pmod{11}, \quad x \equiv 9 \pmod{16}.$$

6. (8 points) Calculate $d(n)$, $\sigma(n)$, and $\phi(n)$ for $n = 2772$.

7. (7 points) Verify that 787 is prime and calculate $\left(\frac{156}{787}\right)$.

8. (5 points) If p is an odd prime, prove that $2 \cdot 4 \cdot 6 \cdots (2p-2) \equiv -1 \pmod{p}$.

9. (10 points) Use the fact that 2 is a primitive root (mod 29) to find all primitive roots (mod 29).

10. (10 points) Solve, if possible, $3x^4 - 12x^2 + 7 \equiv 0 \pmod{47}$.

11. In the Calcrostic below, the letters below represent numbers from the set $\{0, 1, 2, \ldots, 9\}$. Juxtaposition does not mean multiplication, but place value.

$$\begin{array}{ccccc} abb & + & abb & = & cbbb \\ \div & \div & \div & \div & \div \\ d & \times & e & = & f \\ = & = & = & = & = \\ dab & - & cda & = & cda \end{array}$$

(a) (8 points) Solve the Calcrostic.

(b) (7 points) Using the values from the Calcrostic, find the least residue of

$$(a+e)^{abcdef} \pmod{a+b+c+e+d+f}$$