Bishop's University

Math 206: Final Exam
Fall 2013

Name: ____________________

Student #: ____________________

Time: 3 hours

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.

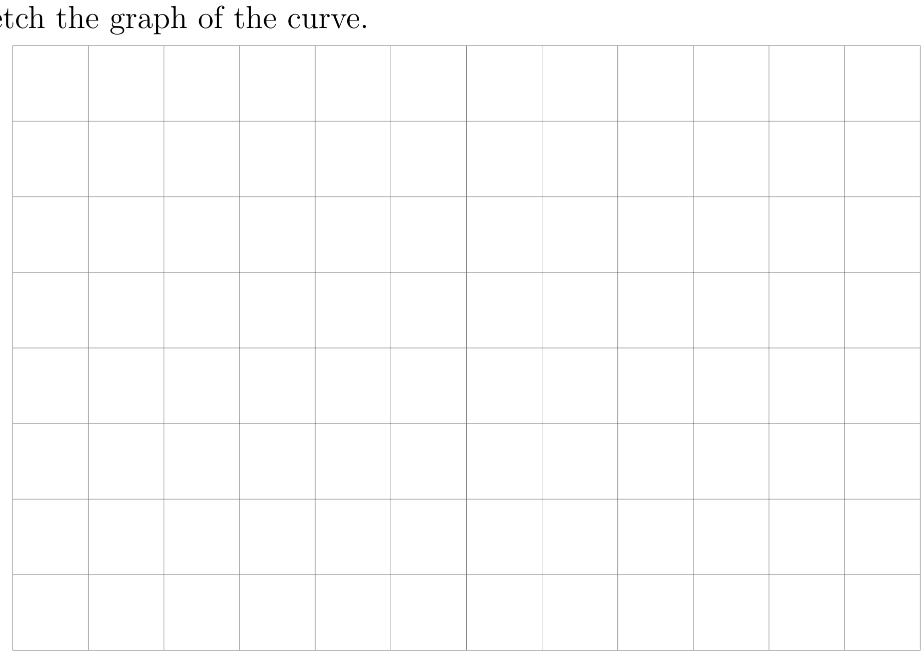
Page	Points	Score
2	16	
3	21	
4	15	
5	12	
6	16	
7	10	
8	10	
Total:	100	

1. (6 points) Find a general equation of the plane containing the points $P(3,1,4)$, $Q(2,7,1)$ and $R(1,4,1)$.

2. Consider the parametric curve given by $x = |t^2 - 4| - 2$, $y = 3 - |t+1|$, for $-3 \leq t \leq 3$.

   (a) (4 points) Find the slope of the tangent line at $t = 1$.

   (b) (6 points) Sketch the graph of the curve.



3. (4 points) Show that the limit $\displaystyle\lim_{(x,y)\to(0,1)} \frac{xy - x}{x^2 + y^2 - 2y + 1}$ does not exist.

4. (9 points) Let $f(x, y, z) = x^3 e^{2yz^2}$. Find all second order partial derivatives.

5. Let $f(x, y) = x^2 - 4xy + 2y^2$.

   (a) (5 points) Find an equation of the tangent plane to the surface $z = f(x, y)$ when $x = -2$ and $y = 2$.

   (b) (3 points) Use part (a) to estimate $f(-2.05, 2.1)$. You may leave the answer in decimal form provided the answer is exact.

6. (5 points) Let $f(x, y) = 3x^2y - 2xy^3$. Find the rate of change of f in the direction of $\vec{u} = \langle 2, -1 \rangle$ at the point $(-1, 1)$.

7. (10 points) Classify the critical points of $f(x, y) = x^3 - 12xy + 8y^3$.

8. (6 points) A right circular cylinder of diameter 6cm and height 10cm is to be painted red over the whole surface. The thickness of the paint is 1mm. Use differentials to estimate the volume of paint used. Recall that the volume of a cylinder is given by $V = \pi r^2 h$.

9. (6 points) Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xy^2$, subject to the constraint $x^2 + y^2 = 16$.

10. (6 points) Evaluate the integral $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} dy dx$ by first reversing the order of integration.

11. (10 points) Use a triple integral to find the volume of the solid enclosed by the paraboloids $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.

12. (10 points) Let E be the solid region bound by the sphere $x^2+y^2+z^2 = 16$ and the cone $z = \sqrt{x^2+y^2}$. Let $f(x, y, z) = x^2+y^2+z^2+1$ be the density at point (x, y, z). Set-up and evaluate the triple integral representing the mass of the solid E. Use spherical coordinates.

13. (10 points) Do **exactly** one of the following parts. Clearly identify the part you want marked.

(a) Find the distance between the lines $\vec{r}_1(t) = \langle 1, -1, 2\rangle + t\langle 4, 3, 1\rangle$ and $\vec{r}_2(t) = t\langle -1, 2, -1\rangle$

(b) Find the radius of the osculating circle to the space curve $\vec{r}(t) = \langle \cos t, \sin t, t^2\rangle$ at the point $(1, 0, \pi^2)$.

(c) Find the absolute maximum and minimum values of $f(x, y) = x^4 - y^4$ on the disk $x^2 + y^2 \leq 4$.