Bishop's University

Math 206: Final Exam
Fall 2013

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	10	
3	8	
4	15	
5	12	
6	20	
7	10	
8	10	
9	15	
10	10	
Total:	110	

1. (10 points) Let $\vec{r}_1(t) = \langle 1, -1, 2\rangle + t\langle 4, 3, 1\rangle$ and $\vec{r}_2(t) = t\langle -1, 2, -1\rangle$ be lines. Find an equation of the plane which is parallel to both $\vec{r}_1$ and $\vec{r}_2$ which passes through the point $P(3, 1, 4)$, and compute the distance between the plane and $\vec{r}_1$.

2. (8 points) Find equations of the tangents to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that pass through the point $(4, 3)$.

3. (5 points) Show that the limit $\lim_{(x,y)\to(0,0)} \dfrac{2xy}{2x^2+y^2}$ does not exist.

4. (10 points) Let $f(x,y) = \dfrac{x+y}{1-xy}$. Find all second order partial derivatives.

5. Let $f(x, y) = 1 + x \ln(xy - 5)$.

   (a) (5 points) Find an equation of the tangent plane to the surface $z = f(x, y)$ when $x = 2$ and $y = 3$.

   (b) (2 points) Use part (a) to estimate $f(2.05, 2.9)$. You may leave the answer in decimal form provided the answer is exact.

6. (5 points) Let $f(x, y, z) = x^2yz - xyz^3$. Find the rate of change of f in the direction of $\vec{u} = \langle 0, 4, -3 \rangle$ at the point $(2, -1, 1)$.

7. (10 points) Classify the critical points of $f(x, y) = x^3 - 3x + 3xy^2$.

8. (10 points) Find the absolute maximum and minimum values of $f(x, y) = x^4 - y^4$ on the disk $x^2 + y^2 \leq 4$.

9. (10 points) Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = xy$, subject to the constraint $x^2 + y^2 = 4$.

10. (5 points) Rewrite the iterated integral $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2) dy dx$ using polar coordinates. **Do NOT evaluate the integral**.

11. (5 points) Rewrite the iterated integral $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} x^2 dz dx dy$ using spherical coordinates. **Do NOT evaluate the integral**.

12. (5 points) Evaluate the integral $\int_0^1 \int_{x^2}^1 \sqrt{y}\sin(y)dydx$ by first reversing the order of integration.

13. (10 points) Let E be the solid region enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$. Let $\rho(x, y, z) = x^2 + y^2$ be the density at point (x, y, z). Set-up and evaluate the triple integral representing the mass of the solid E.

14. (10 points) Evaluate the iterated integral

$$\iint_R \frac{x-2y}{3x-y} dA,$$

where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, and $3x-y=8$ by first making an appropriate change of variables.