Bishop's University

Math 206: Final Exam
Fall 2023

Name: ______________________________

Student #: ______________________________

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	13	
3	13	
4	13	
5	10	
6	10	
7	16	
8	10	
9	15	
Total:	100	

1. (5 points) Find a general equation of the plane containing the points $P(1,5,9)$, $Q(8,2,8)$ and $R(1,6,1)$.

2. Consider the parametric curve given by $x = \sec t$, $y = \sin t$, for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

   (a) (4 points) Find the slope of the tangent line at $t = -\frac{\pi}{4}$.

   (b) (4 points) Sketch the graph of the curve.

3. (4 points) Show that the limit $\lim_{(x,y)\to(0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$ does not exist.

4. (9 points) Let $f(x, y, z) = z\sin(xy + 3z)$. Find all second order partial derivatives.

5. Let $f(x, y) = x \sin(x + y)$.
   (a) (5 points) Find an equation of the tangent plane to the surface $z = f(x, y)$ when $x = -1$ and $y = 1$.

   (b) (3 points) Use part (a) to estimate $f(-1.05, 1.1)$. You may leave the answer in decimal form provided the answer is exact.

6. (5 points) Let $f(x, y) = \dfrac{x}{x^2 + y^2 + 1}$. Find the rate of change of f in the direction of $\vec{u} = \langle 3, 5 \rangle$ at the point $(1, 2)$.

7. (10 points) Classify the critical points of $f(x, y) = 3x - x^3 - 2y^2 + y^4$.

8. (5 points) A right circular cylinder of diameter 10cm and height 6cm is to be painted red over the whole surface. The thickness of the paint is 1mm. Use differentials to estimate the volume of paint used. Recall that the volume of a cylinder is given by $V = \pi r^2 h$.

9. (5 points) Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y) = 2x^2 + 6y^2$, subject to the constraint $x^4 + 3y^4 = 1$.

10. (6 points) Evaluate the integral $\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x}\, dx dy$ by first reversing the order of integration.

11. (10 points) Use a triple integral to find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$.

12. (10 points) Let E be the solid region bound by the sphere $x^2+y^2+z^2=9$ and the cone $z=\sqrt{x^2+y^2}$. Let $f(x,y,z)=x^2+y^2+z^2+1$ be the density at point (x,y,z). Set-up and evaluate the triple integral representing the mass of the solid E. Use spherical coordinates.

13. (10 points) Find the area of the region that lies inside the polar curve $r = 1 + \cos\theta$ and outside the polar curve $r = 2 - \cos\theta$.

14. (5 points) Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.