Bishop's University

Math 207: Final Exam
Winter 2021

Name:	____________________
Student #:	____________________

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	10	
3	10	
4	10	
5	5	
6	10	
7	15	
8	5	
9	20	
10	15	
Total:	100	

1. (10 points) Without reference to a potential function, show that $\mathbf{F}(x,y) = \langle e^x \sin y, e^x \cos y + \sin y \rangle$ is conservative and find a function ϕ such that $\mathbf{F} = \nabla \phi$. Use ϕ to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the arc of an ellipse going from $(0,0)$ to $(-1, \frac{\pi}{4})$.

2. (10 points) Use Green's Theorem to find $\oint_C (y^3 \, \mathrm{d}x - x^3 \, \mathrm{d}y)$ where C is the circle $x^2 + y^2 = 4$ travelled counterclockwise.

3. For this question, let $\mathbf{F} = \langle x^2yz, xy^2z, xyz^2 \rangle$.
   (a) (8 points) Find the divergence and curl of $\mathbf{F}$.
   (b) (2 points) Is it possible to express $\mathbf{F}$ of part (a) as the gradient of a function f?

4. (5 points) Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

5. For this question, **SET-UP TO THE POINT OF EVALUATION BUT DO NOT EVALUATE THE INTEGRALS.**

   (a) (5 points) Use Stokes' Theorem to write $\oint_C \mathbf{F} \cdot d\mathbf{r}$ as a double integral, where $\mathbf{F} = \langle 3, z^2, yz \rangle$ and C is the boundary of the paraboloid $y = 4 - x^2 - z^2$ in the first octant travelled clockwise as viewed from the origin.

   (b) (5 points) Use the Divergence Theorem to write the flux of $\mathbf{F} = 2x^3z\mathbf{i} + 2y^3z\mathbf{j} + 3z^2\mathbf{k}$ across the sphere $x^2 + y^2 + z^2 = 4$, oriented outward, as a triple integral.

6. Test the following series for convergence or divergence:

   (a) (5 points) $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k^2}$

   (b) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{2^{2n+1}}$

   (c) (5 points) $\displaystyle\sum_{i=1}^{\infty} \frac{e^{\frac{1}{i}}}{i^2 + i}$

7. (5 points) Determine, with justification, whether the following series is absolutely convergent, conditionally convergent or divergent: $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-2}$

8. Let $f(x) = \sum_{n=0}^{\infty} (n+1)\frac{x^n}{3^n}$.

   (a) (5 points) Find the radius of convergence for f

   (b) (10 points) Find a series for $\int f(x)\,\mathrm{d}x$ and a formula for its sum. **(Hint: geometric series)**

   (c) (5 points) Find a formula for $f(x)$. **(Hint: $f(x)$ is the derivative of $\int f(x)\,\mathrm{d}x$)**

9. (15 points) Let $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$. Find the arclength function for this curve, starting at $t = 0$ moving in the direction of increasing t. Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$, and κ at $t = 1$ and determine the center of curvature at this point.

This page is attached for extra work.