Bishop's University

Math 207: Final Exam
Winter 2024

Name:	
Student #:	

- This exam is 180 minutes in length.
- All solutions must be written on this exam paper. No extra paper is permitted.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	10	
4	10	
5	15	
6	10	
7	10	
Total:	75	

1. (5 points) Evaluate the line integral, $\int_C yz \cos x ds$, where C is the curve $\vec{r}(t) = \langle t, 3\cos t, 3\sin t\rangle$ with $0 \leq t \leq \pi$.

2. Let $\vec{F} = \langle (1+xy)e^{xy}, e^y + x^2 e^{xy}\rangle$ be a vector field.

   (a) (5 points) Without reference to a potential function, show that $\vec{F}$ is a conservative vector field.

   (b) (5 points) Find a function f such that $\vec{F} = \nabla f$.

   (c) (5 points) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve $\vec{r}(t) = \langle t + \sin(\pi t), 2t + \cos(\pi t)\rangle$ for $0 \leq t \leq 1$.

3. (5 points) Use Green's Theorem to find the area bounded by the ellipse with parametric equation $\vec{r}(t) = \langle 3\cos t, 2\sin t \rangle$.

4. (5 points) Use the Divergence Theorem to calculate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F}(x, y, z) = \langle x^3, y^3, z^3 \rangle$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.

5. For the curve given by $\vec{r}(t) = \langle \sin^3 t, \cos^3 t, \sin^2 t \rangle$, $0 < t < \dfrac{\pi}{2}$ find,

(a) (2 points) the unit tangent vector.

(b) (3 points) the unit normal vector.

(c) (2 points) the unit binormal vector.

(d) (3 points) the curvature.

6. (5 points) Determine whether the sequence $a_n = \dfrac{n^2 + \cos n}{3n^2 + 2}$ is convergent or divergent. If it is convergent, find its limit.

7. (5 points) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+1) 3^n}{2^{2n+1}}$ is absolutely convergent, conditionally convergent, or divergent.

8. (5 points) For what values of x does the series $\displaystyle\sum_{n=1}^{\infty} (\ln x)^n$ converge?

9. (5 points) How many terms of the series are necessary to estimate the sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ to within 0.00001 of it actual value.

10. (5 points) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{(n+2)!}$.

11. (5 points) Find the first five (5) non-zero terms of the Taylor series for $f(x) = \sin x$ at $a = \dfrac{\pi}{6}$. **No factorials are permitted in the final answer.**

12. (5 points) Find the Taylor polynomial of degree 2 for $f(x) = \sec x$ at $a = 0$, and estimate the accuracy of the approximation $f(x) = T_2(x)$ for the interval $[0, \pi/6]$.