Bishop's University

Math 217: Final Exam
Winter 2013

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Unless directed otherwise, express your answers in the form $x+iy$.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*

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1. (5 points) Show that $\lim_{z\to 0} \frac{z}{\overline{z}}$ does not exist.
2. (15 points) Express in the form $a+bi$, $a, b \in \mathbb{R}$.
   (a) $(1+i)^{17}$.
   (b) All solutions to $z^2+2z+1-i=0$.
   (c) All solutions to $\sin z = -3i$.
3. (4 points) If $z = x+iy$, for what x and y are all values of 1^z real?
4. (4 points) For what values of z is it true that $\text{Ln } z^2 = 2\text{Ln } z$.
5. (9 points) State the Cauchy-Riemann equations and **answer two of the following three parts**:
   (a) Verify that the Cauchy-Riemann equations are satisfied for the function $f(z) = \sin 2z$.
   (b) Show that if the function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, show that the function
   $$g(z) = \overline{f(1-\overline{z})}$$
   is also analytic in D.
   (c) Show that if $\phi(x,y)$ is harmonic in a domain D, then $f(z) = \phi_x(x,y) - i\phi_y(x,y)$ is analytic in D.
6. (4 points) Classify the singular points of $\frac{z^3+1}{z^2(z+1)}$.
7. (8 points) Find all Laurent series for
   $$f(z) = \frac{z}{(z+1)(z-2)}$$
   about $z=0$.
8. (7 points) Evaluate $\oint_C \frac{\cosh z^2}{z(z^2+4)}$ where C is
   (a) $|z-i| = 2$, travelled counterclockwise.
   (b) $|z-2| = 1$, travelled clockwise.
9. (5 points) Compute $\oint_C \frac{1}{z \sin z} dz$ where C is the curve $|z| = 1$ travelled counterclockwise.
10. (6 points) Find the principal value of $\int_0^\infty \frac{x^2+1}{x^4+1} dx$
11. (10 points) Consider the linear fractional transformation that sends
    $$-1 \text{ to } -1$$
    $$i \text{ to } 0$$
    $$1 \text{ to } 1.$$
    (a) Find the transformation.
    (b) What does the interior of $|z| = 1$ get mapped to?
    (c) What are the fixed points?