Bishop's University

Math 310/Physics 270: Final Exam
Fall 2016

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** calculator is permitted. No other electronic calculators are permitted.

Page	Points	Score
2	20	
3	20	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	90	

1. (10 points) Find the general solution of $t^2 + 4y + t\dfrac{dy}{dt} = 0$.

2. (10 points) Solve the initial value problem $7xy - 3(\sqrt{1-x^2})y' = 0$, $y(0) = -5$ and state the domain of definition.

3. (10 points) Show that $x^2y^3 + x(1+y^2)y' = 0$ becomes exact when multiplied by the integrating factor $\mu(x,y) = \dfrac{1}{xy^3}$. Solve the equation with the initial condition $y(4) = 1$.

4. (10 points) Use the method of undetermined coefficients to find the general solution of

$$y'' - 6y' + 13y = 7\sin t.$$

5. (10 points) Use the method of variation of parameters to solve

$$y'' - \frac{5}{2}y' - 6y = \frac{3}{2}e^t, \quad y(0) = 1, \quad y'(0) = 0.$$

6. (10 points) Find the general solution of

$$y''' - 3y'' + 4y = -8e^{-3t}$$

7. (a) (5 points) Write $t^2\dfrac{d^3y}{dt^3} - 11t\dfrac{d^2y}{dt^2} + 30\dfrac{dy}{dt} - 12y = 80e^t$, $y(1) = -2$, $y'(1) = 3$, $y''(1) = 1$, as a system of first order equations.

(b) (5 points) Write the system of equations

$$\vec{\mathbf{x}}' = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \vec{\mathbf{x}}$$

as a single second order equation.

8. (10 points) Solve:

$$\vec{\mathbf{x}}' = \begin{bmatrix} 2 & -2 & 2 \\ -3 & 5 & 0 \\ -15 & 22 & -5 \end{bmatrix} \vec{\mathbf{x}}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}.$$

9. (10 points) A mass weighing 10 pounds stretches a spring 6 inches. The mass is then displaced 1 foot downward and then released. Formulate the initial value problem describing the motion of the mass and solve.