Bishop's University

Math 310/Physics 270: Final Exam
Fall 2017

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Do not remove any pages from this test.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other electronic calculators are permitted.

Page	Points	Score
2	20	
3	20	
4	10	
5	10	
6	10	
7	15	
8	15	
Total:	100	

1. (10 points) Find the general solution of $\dfrac{dy}{dt} = 3 - 6t + y - 2ty$.

2. (10 points) Solve the initial value problem $2y + xy' = \frac{\sin(x)}{x}$, $y(2) = 1$ and state the domain of definition.

3. (10 points) Show that $y + \left(2x - \dfrac{e^y}{y}\right) y' = 0$ becomes exact when multiplied by the integrating factor $\mu(x, y) = y$. Solve the equation with the initial condition $y(4) = 1$.

4. (10 points) Use the method of undetermined coefficients to find the general solution of

$$y'' + 2y' + y = 2e^{-t}.$$

5. (10 points) Use the method of variation of parameters to solve

$$y'' - y' - 2y = 2e^{-t}, \quad y(0) = 1, \quad y'(0) = 0.$$

6. (10 points) Find the general solution of

$$y''' - y'' + y' - y = \sin(2t)$$

7. (a) (5 points) Write $\dfrac{d^3y}{dt^3} - 11t\dfrac{d^2y}{dt^2} + 30t^2\dfrac{dy}{dt} - 12t^3y = 80\cos(t)$, $y(0) = -2$, $y'(0) = 0$, $y''(0) = 1$, as a system of first order equations.

(b) (5 points) Write the system of equations

$$\vec{\mathbf{x}}' = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \vec{\mathbf{x}}$$

as a single second order equation.

8. (15 points) Solve, and write the solution in terms of real-valued functions:

$$\vec{\mathbf{x}}' = \begin{bmatrix} 2 & 0 & -3 \\ 0 & -2 & 1 \\ 0 & -1 & -2 \end{bmatrix} \vec{\mathbf{x}}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}.$$

9. (15 points) A tank initially contains 120 L of pure water. A mixture containing a concentration of 3 g/L of salt enters the tank at a rate of 2L/min, and the well-stirred mixture leaves the tank at the same rate. Formulate the initial value problem describing the amount of salt in the tank and solve.