Bishop's University

Math 310/Physics 270: Final Exam
Fall 2018

Name: _____

Student #: _____

- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- The back of each page may be used for scrap paper.
- A **Casio fx260-solar** or **Casio fx260-solar II** calculator is permitted. No other aids are permitted.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	20	
3	20	
4	10	
5	10	
6	10	
7	15	
8	15	
Total:	100	

1. (10 points) Find the general solution of $ty + y - t^2y' = 0$ and state all possible domains of definition.

2. (10 points) Solve the initial value problem $xy' + 3y = x^{3/2}$, $y(4) = 1$ and state the domain of definition.

3. (10 points) Solve the initial value problem $t^2y'' - 3ty' + 4y = 0$, $y(1) = 1$, $y'(1) = 0$.

4. (10 points) Use the method of undetermined coefficients to find the general solution of

$$y'' + 2y' + y = 2e^t.$$

5. (10 points) Use the method of variation of parameters to solve

$$y'' + 3y' + 2y = 2e^{2t}, \quad y(0) = 1, \quad y'(0) = 0.$$

6. (10 points) Find the general solution of

$$y''' - 2y'' + y' - 2y = e^t \cos(2t)$$

7. (a) (5 points) Write $\dfrac{d^3y}{dt^3} - 12\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} - 13y = 0$, $y(0) = -5$, $y'(0) = 1$, $y''(0) = 2$, as a system of first order equations.

(b) (5 points) Write the system of equations, with initial values,

$$\vec{\mathbf{x}}' = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \vec{\mathbf{x}}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

as a single second order equation, with initial values.

8. (15 points) Solve, and write the solution in terms of real-valued functions:

$$\vec{\mathbf{x}}' = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \vec{\mathbf{x}}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

9. (15 points) A tank initially contains 100 L of salt water with a concentration of 3g/L. A mixture containing a concentration of 12 g/L of salt enters the tank at a rate of 5L/min, and the well-stirred mixture leaves the tank at the same rate. Formulate the initial value problem describing the amount of salt in the tank and determine the exact length of time it takes for the amount of salt in the tank to reach 1kg (1000g).