Bishop's University

Math 310/Physics 270: Final Exam
Fall 2023

Name: _____

Student #: _____

- This test is 180 minutes in length.
- All answers must be exact (no decimals allowed) unless specifically directed otherwise.
- Prepare neat solutions. Briefly justify your work, that is, *make your reasoning clear.*
- You are permitted to use one (1) **Authorized Memory Book** and a **Casio fx-260 Solar (II) calculator**.
- Do not remove any pages from this test.
- All answers must be written in the space provided.
- The back of each page may be used for scrap paper.
- **Remember that Bishop's University has a ZERO-TOLERANCE POLICY for academic misconduct on final exams.**

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	15	
9	15	
10	10	
Total:	100	

1. (10 points) Find the general solution of $xyy' = x^2 + y^2$, $x < 0$.

2. (10 points) Solve the initial value problem $(x-1)^2y' + 3(x-1)y = (x-1)^{3/2}$, $y\left(\frac{3}{2}\right) = 1$ and state the domain of definition (that is, where this solution is valid).

3. (10 points) Solve the initial value problem $y'' - \frac{3}{t}y' + \frac{4}{t^2}y = 0$, $y(1) = 0$, $y'(1) = -1$.

4. (10 points) Use the method of undetermined coefficients to find the general solution of

$$y''' + 2y'' - y' - 2y = \sin(2t)$$

5. (10 points) Use the method of variation of parameters to solve

$$y'' + 3y' + 2y = e^{2t}, \quad y(0) = 0, \quad y'(0) = 1.$$

6. (10 points) A spring with a 2-kg mass can be held stretched 0.2 meters beyond its natural length by a force of 80 newtons (kg·m/sec^2). If the spring begins at its equilibrium position, but a push gives it an initial velocity of 2 m/sec, find the position of the mass after t seconds.

7. (15 points) Solve using matrix exponentials, and write the solution in terms of real-valued functions:

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

8. (15 points) A tank initially contains 100 L of salt water with a concentration of 3g/L. A mixture containing a concentration of 10 g/L of salt enters the tank at a rate of 5L/min, and the well-stirred mixture leaves the tank at the same rate. Formulate the initial value problem describing the amount of salt in the tank and determine the exact length of time it takes for the amount of salt in the tank to reach 900g.

9. (10 points) Write the system of equations, with initial values,

$$\vec{\mathbf{x}}' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{x}}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

as a single third order equation, with initial values. **DO NOT SOLVE THE SYSTEM OR THE THIRD ORDER EQUATION!**